STUDY OF VIBRATION AND AUDIBLE NOISE GENERATED BY SHUNT CAPACITOR BANKS AT SUBSTATIONS SERVING NON-LINEAR LOADS

by

ADAM BRENT ROGET

TIMOTHY A. HASKEW, COMMITTEE CHAIR
SHUHUI LI
WILLIAM S. SHEPARD
KEITH A. WOODBURY

A THESIS

Submitted in partial fulfillment of the requirements for the degree of Master of Science in the Department of Electrical Engineering in the Graduate School of The University of Alabama

TUSCALOOSA, ALABAMA

2011
ABSTRACT

At the request of a local utility provider, a study was conducted to determine why capacitor banks located at two substations supplying power to a steel mill are creating audible noise. A method to decrease the amount of audible noise was also requested. The construction of the banks and the possible cause of excitation are discussed. Electrical current and acoustic harmonic measurements taken at each substation are presented. Vibration testing of the banks was conducted to determine the mechanical natural frequencies, and the results are given. Possible solutions to reduce the amount of audible noise from the capacitors conclude the discussion.
LIST OF ABBREVIATIONS AND SYMBOLS

\( a \)  
Length of the plate

\( A \)  
Surface area of conducting plate

\( b \)  
Width of the plate

\( C \)  
Capacitance

\( d \)  
Dielectric film thickness

\( E \)  
Modulus of elasticity

\( \varepsilon_0 \)  
Dielectric constant of vacuum

\( \varepsilon_1 \)  
Dielectric film relative dielectric constant

\( \varepsilon_2 \)  
Dielectric fluid relative dielectric constant

\( f \)  
Generated force

\( FEA \)  
Finite element analysis

\( \gamma \)  
Mass per unit area

\( h \)  
Plate thickness

\( i \)  
Current (Chapter 3)

\( i \)  
Number of half waves along the horizontal axis (Chapter 4 and 5)

\( j \)  
Number of half waves along the vertical axis

\( p \)  
Power

\( p_r \)  
Generated pressure
\( p_{\text{max}} \) Maximum generated pressure
\( \rho \) Density
\( q \) Charge
\( \text{RMS} \) Root mean square difference
\( \text{SUB 1} \) First transmission substation
\( \text{SUB 2} \) Second transmission substation
\( \nu \) Poisson’s ratio
\( \nu \) Time dependent voltage
\( V \) Constant voltage
\( W \) Capacitor electrical energy storage
\( x \) Distance along the width of the plate
\( X \) Mode shape in x direction
\( y \) Distance along the length of the plate
\( y \) Plate separation (Chapter 2)
\( Y \) Mode shape in y direction
\( Z \) Complete mode shape
ACKNOWLEDGEMENTS

I would like to thank my entire committee for all of their help and encouragement. I would especially like to thank Dr. Keith Woodbury for funding my graduate study at The University of Alabama. Also, this work would have not been possible without the guidance and equipment given by Dr. William S. Shepard and Keith Williams at The University of Alabama. Finally, I would like to thank my friends and family for their support during my graduate study.
CONTENTS

ABSTRACT ........................................................................................................................................... ii

LIST OF ABBREVIATIONS AND SYMBOLS ....................................................................................... iii

ACKNOWLEDGEMENTS ........................................................................................................................... v

LIST OF TABLES ........................................................................................................................................ viii

LIST OF FIGURES ...................................................................................................................................... ix

CHAPTER 1: INTRODUCTION .................................................................................................................. 1

CHAPTER 2: PREVIOUS WORK AND PROJECT GOALS ......................................................................... 5

Conclusions ................................................................................................................................................ 12

Goals of this Study ................................................................................................................................... 13

CHAPTER 3: CAPACITOR DISSECTION AND ANALYSIS ....................................................................... 15

Literature Review ..................................................................................................................................... 16

Dissection .................................................................................................................................................. 22

Analysis .................................................................................................................................................... 28

Non-Linear Excitation ............................................................................................................................... 32

Conclusions ............................................................................................................................................... 35

CHAPTER 4: SHELL MODELING ............................................................................................................. 37

Blevins’s Methods ................................................................................................................................... 37
FEA Analysis ........................................................................................................................................... 44
Conclusions .............................................................................................................................................. 55

CHAPTER 5: VIBRATION ANALYSIS ........................................................................................................ 57
Setup ....................................................................................................................................................... 58
Testing ..................................................................................................................................................... 64
Validation ............................................................................................................................................... 85
Results ..................................................................................................................................................... 88
Conclusions ........................................................................................................................................... 89

CHAPTER 6: CONCLUSIONS AND POSSIBLE SOLUTIONS ................................................................. 91
Possible Solutions ..................................................................................................................................... 93
  Filters ..................................................................................................................................................... 94
  Shifting Mechanical Natural Frequencies ............................................................................................. 95
  Acoustic Enclosures .............................................................................................................................. 95
  Redesign the Capacitor ........................................................................................................................ 96
Recommended Solution .......................................................................................................................... 96
Future Work ............................................................................................................................................. 97
REFERENCES ........................................................................................................................................... 98
LIST OF TABLES

Table 1. Element Layers Measured by McDuff .................................................. 22
Table 2. Calculated Natural Frequencies for Broad Side with Simple Conditions .......... 43
Table 3. Calculated Natural Frequencies for Narrow Side with Simple Conditions .......... 43
Table 4. Calculated Natural Frequencies for Broad Side with Clamped Conditions .......... 44
Table 5. Calculated Natural Frequencies for Narrow Side with Clamped Conditions .......... 44
Table 6. FEA Convergence for the Free Plate .................................................. 45
Table 7. FEA Calculated Natural Frequencies for the Shell Without the Broad Side .......... 49
Table 8. FEA Calculated Natural Frequencies for the Complete Shell ......................... 53
Table 9. Free Plate Natural Frequency Comparison ........................................ 68
Table 10. Initial Test Locations for the Broad Side ........................................... 71
Table 11. Structured Excitation Locations for the Broad Side ................................ 73
Table 12. Narrow Side Excitation Locations ..................................................... 77
Table 13. Sine and Sweep Response Comparison ............................................. 86
Table 14. Airborne Sound Transmission Loss Through Composite Acoustic Enclosure .... 96
Table 15. Estimated Noise Reduction with Enclosure ....................................... 97
LIST OF FIGURES

Figure 1. Measured Capacitor Current Waveform ................................................................. 6
Figure 2. FFT of Measured Capacitor Current ................................................................. 7
Figure 3. System Frequency Response with Bank at SUB 1 Online ........................................... 8
Figure 4. System Frequency Response with Bank at SUB 2 Online ........................................... 9
Figure 5. System Frequency Response with Both Banks Online ............................................ 9
Figure 6. SUB 1 Current and Acoustic Spectrum .............................................................. 10
Figure 7. SUB 2 Acoustic Spectrum .............................................................................. 11
Figure 8. Typical Shunt Capacitor Construction with Elements Vertically Arranged [9] .......... 17
Figure 9. Parallel Plate Capacitor with Variable Plate Separation [13] ................................. 18
Figure 10. Forces Generated Between Two Charged Plates ............................................ 19
Figure 11. Voltage and Voltage Squared Waveform Comparison .......................................... 20
Figure 12. Typical Relaxed Layer of a Wound Capacitor [13] ........................................... 21
Figure 13. Direction of Forces Generated by the Capacitive Elements ................................. 22
Figure 14. Failed Capacitor with Broad-Side Plate Removed ........................................... 23
Figure 15. Failed Capacitor with Kraft Paper Peeled Back ................................................ 24
Figure 16. Observed Point of Failure .................................................................................. 24
Figure 17. Internal Structure of the Failed Capacitor ......................................................... 25
Figure 18. Capacitive Element with Recorded Dimensions ............................................ 26
Figure 19. Layers of the Capacitive Elements ........................................................................ 26
Figure 20. Observed Element Connections ........................................................................ 27
Figure 21. Equivalent Element Connections ...................................................................... 27
Figure 22. Direction of Forces for Wound Section ............................................................... 30
Figure 23. Calculated Maximum and Minimum Pressure Generation: Broad Side .............. 32
Figure 24. Estimated Voltage at SUB 1 Bank ...................................................................... 34
Figure 25. FFT of the Voltage Squared Waveform ............................................................... 35
Figure 26. (a) Simply Supported and (b) Clamped End Conditions ...................................... 38
Figure 27. Modes Shapes for a Plate Simply Supported on All Ends .................................... 40
Figure 28. Mode Shapes for a Beam with Simply Supported and Clamped Ends [4] .......... 41
Figure 29. Mode Shapes for a Free Plate: 396 Hz to 2,170 Hz ............................................ 45
Figure 30. Mode Shapes for a Free Plate: 2,487 Hz to 3,425 Hz .......................................... 46
Figure 31. Shell Without Broad Side for FEA Analysis ....................................................... 47
Figure 32. FEA Mode Shapes for the Shell Without the Broad Side: 60 Hz to 663 Hz ........ 49
Figure 33. FEA Mode Shapes for the Shell Without the Broad Side: 776 Hz to 1,495 Hz .... 50
Figure 34. FEA Mode Shapes for the Shell: 61 Hz to 662 Hz ............................................. 53
Figure 35. FEA Mode Shapes for the Shell: 836 Hz to 1,440 Hz ........................................ 54
Figure 36. Typical Scanning Laser Vibrometer Setup ......................................................... 59
Figure 37. Scanning Laser Vibrometer System Components [16] ...................................... 60
Figure 38. Experimental Excitation Setup .......................................................................... 61
Figure 39. Experimental Measurement Setup ...................................................................... 61
Figure 40. Laser Vibrometer Position ................................................................................ 62
Figure 41. Free Plate Test Setup ......................................................................................... 65
Figure 65. Experimental Mode Shapes for the Narrow Side of the Shell .................................. 84
Figure 66. Sine and Sweep Response Comparison ............................................................... 86
Figure 67. Comparison of the Fast Sweep and Slow Sweep Response ................................... 87
Figure 68. Comparison of the Average Response for the Narrow Side and Broad Side .......... 88
Figure 69. Increase in Frequency Response for the Broad Side ............................................. 89
Figure 70. Change in System Electrical Resonance Resulting from Capacitor Switching ....... 94
CHAPTER 1
INTRODUCTION

To reduce the reactive power demand from a steel mill and improve voltage regulation, the mill’s utility provider has a large bank of capacitors installed at each of the two substations supplying power to the mill. For melting scrap metal, the mill uses a DC electric arc furnace powered by a 12 pulse rectifier. If either of the two capacitor banks are online while the mill is melting scrap metal, audible noise is generated by the banks. One of the substations is located near a neighborhood, and several complaints have been received regarding the audible noise. The utility provider is interested in determining the root cause of the audible noise and also finding a solution to reduce the amount of noise created by the banks.

Audible noise generated by a capacitor has also been noticed on other occasions. Recently, it was observed by the author at a substation that fed a large hoist motor at another local company. For controlling the hoist speed, a DC motor is used that is powered by a 12 pulse rectifier. The noise generated at the substation was severe enough that the relative position of the skip could be inferred from the changes in acoustic frequency. Another incident was noted by Cox and Guan [5] who performed a similar study on the acoustic generation by shunt capacitors at a substation that fed an electric arc furnace. Their study was a great source of information for this investigation and is discussed throughout the paper.
The one thing all of these occurrences have in common is they all were documented at substations that fed DC equipment supplied through rectification. When the equipment is in operation, current at harmonics of the fundamental is injected into the system. This is a result of the non-linear operation of the power switching devices. As the cost of power electronics continues to decrease, their use in industry will continue to grow. This will likely cause an increase in instances of acoustic noise generated by capacitors.

In an effort to find a solution to reduce the amount of noise generated, the local utility provider contacted Dr. Timothy Haskew, the director of this research project. At the outset of this investigation, Haskew [11] collected and processed electrical current and acoustic data from the substations, which is discussed in detail in Chapter 2. After analyzing the data, it was discovered that the data had characteristics that were unexpected. This study seeks to resolve issues uncovered through the previous analysis.

One of the primary goals of this study was to determine the root cause of the noise generation. A failed shunt capacitor, similar to those used at both substations, was donated by the utility provider for dissection. This is discussed in Chapter 3. Through the dissection and subsequent analysis, it was determined that electric forces generated within the capacitive elements act to constrict them. When the elements return to their relaxed state, the shell of the capacitor is either excited through direct contact with the elements or pressure waves generated by the elements within the liquid dielectric. Estimates for these forces are calculated and their non-linear behavior is explained.

In Chapter 2, data is presented that shows that, in some cases, the largest acoustic responses occur at frequencies of the least dominant harmonic currents. This leads to the conclusion that amplification through mechanical resonances is occurring. The natural
frequencies of the shell, as well as their corresponding mode shapes, are estimated using theoretical methods in Chapter 4. Limits for the natural frequencies are determined by considering the shell as a series of plates with different end conditions. FEA analysis is then used to determine the natural frequencies and mode shapes of the complete shell. Because of the complexities of the complete system and time limitations, the entire capacitor was not modeled using theoretical methods. However, the analysis presented serves as a starting point for further modeling.

The majority of the effort spent on this project was used to experimentally determine the natural frequencies of a shunt capacitor typical of those used at both substations. This is discussed in Chapter 5. For the vibration analysis, a scanning laser vibrometer was used to determine the natural frequencies of the whole capacitor, the capacitor with no liquid dielectric, and the shell of the capacitor. Through the analysis, it was determined that the broad sides have peaks in the response near harmonics of 60 Hz. Analyzing the capacitor with no liquid dielectric showed that the dielectric has a large damping effect on the response of the system. The scanning laser vibrometer has the ability to also determine mode shapes experimentally, but because the system is heavily damped, no standing mode shapes were seen. The shell was analyzed mainly to have a comparison for the theoretical model. Overall, the experimental analysis of the shell matched the theoretical methods well, as the measured natural frequencies and mode shapes came within 10% of the theoretical results.

To conclude the thesis, possible solutions to reduce the audible noise generated at the substations were investigated. Solutions that were considered include filtering the harmonics, changing the system natural frequencies, re-designing the capacitors, and creating an acoustic
enclosure around each capacitor. It was determined that enclosing the capacitor is the best option to reduce the magnitude of the audible noise.
CHAPTER 2
PREVIOUS WORK AND PROJECT GOALS

When exposed to current harmonics injected into the system by an electric arc furnace, banks of capacitors at two transmission substations generate a great deal of audible noise. In an effort to reduce the negative effects, a solution to reduce the amount of noise generated by the banks was sought. The task of investigating the audible noise problem was first undertaken by Haskew at The University of Alabama [11]. The original goals of the project were to perform a literature search to determine the root cause of the audible noise, develop a model of the system in the Electromagnetic Transients Program- Restructured Version (EMTP-RV), perform field measurements at the substations, and determine a solution to decrease the audible noise. What follows in this chapter is a synopsis of Haskew’s work.

At the first substation (SUB 1), the capacitor bank is rated at 15 MVAR and at the second substation (SUB 2), the capacitor bank is rated at 28.8 MVAR. Both banks operate nominally at 115 kV. The mill’s 230 kV bus is fed using two 230:115 kV autobank transformers located at SUB 2. The mill is connected to the 230 kV bus through two 230:34.5 kV transformers. This bus supplies power to the mill’s 130 MVA DC electric furnace, which is powered by a 12 pulse AC/DC converter. To help reduce power quality issues, the furnace is equipped with a 0 to +100 MVAR static VAR compensator, which is connected to the
34.5 kV bus. The compensator is composed of a thyristor controlled reactor and five filter banks (the 2\textsuperscript{nd}, 3\textsuperscript{rd}, 4\textsuperscript{th}, 5\textsuperscript{th}, and 7\textsuperscript{th} harmonic of 60 Hz)

For the study, current data for the capacitor bank was gathered by the utility provider at SUB 1. Figure 1 shows the current measured at each phase while the audible noise was generated. The sampling rate for the data was approximately 15 kHz. From the figure, some distortion is visible in the peaks of the waveform. Figure 2 shows the FFT of the three currents. For all frequency plots given in this study, the first harmonic is at 60 Hz. The FFT shows current spikes at the odd harmonics of 60 Hz. The dominant peaks occur at the 5\textsuperscript{th}, 7\textsuperscript{th}, 11\textsuperscript{th}, 13\textsuperscript{th}, 23\textsuperscript{rd}, and 25\textsuperscript{th} harmonic of 60 Hz. Current injection at the 11\textsuperscript{th}, 13\textsuperscript{th}, 23\textsuperscript{rd}, and 25\textsuperscript{th} harmonics is typical of 12 pulse rectifiers while injections at the 5\textsuperscript{th} and 7\textsuperscript{th} harmonics are typical of 6 pulse rectifiers, but are often also present with 12 pulse rectifiers.

![Figure 1. Measured Capacitor Current Waveform](image-url)
With a system model created in EMTP-RV by the utility provider, Haskew began the project by determining the system’s electrical resonance frequencies. Using the model, he first simulated taking the capacitor banks offline, injecting current at the mill, and sweeping the frequency to 3 kHz. The frequency response of various monitored voltages and currents were simulated. He determined that the resonance frequencies of the system without the capacitor banks online occur at center frequencies of 640 Hz and 2,800 Hz. Using the same model, simulations were run after bringing the capacitor banks at SUB 1 and SUB 2 online, individually, and together. Again, current was injected at the mill and the frequency was swept to 3 kHz. The frequency response of the system with the only the bank at SUB 1 online is shown in Figure 3. With only the bank at SUB 1 online, the system electrical resonances occur at approximately 270 Hz, 360 Hz, 470 Hz, 580 Hz, 1,385 Hz, and 1,445 Hz. The resonances do not occur exactly at harmonic frequencies, but some are close. The lower resonances (580 Hz and below) are near
even harmonics, which are of minimal concern. There are resonances near the 23\(^{rd}\) and 25\(^{th}\) harmonics, two of the frequencies of injection for the 12 pulse rectifier. Figure 4 shows the frequency response of the system with only the bank at SUB 2 online. The lower frequency resonances remain almost the same as in Figure 3, but the dominant resonant frequency moves to 1,110 Hz. Figure 5 shows the frequency response of the system with both of the banks online. With both banks online, the dominant resonant frequencies move to 1,075 Hz and 1,825 Hz, while the lower frequency resonances are relatively unaffected.

In addition to performing the EMTP simulations, Haskew collected field measurements at both of the substations. Using an instrument microphone and oscilloscope with a built in FFT module, he recorded sound pressure levels at each substation while the furnace was operating. The utility provider simultaneously recorded current through the bank at SUB 1. One phase was monitored and the fundamental and harmonic RMS current through the banks connected to that phase were sampled every half second for 15 minutes. The RMS values

![Figure 3. System Frequency Response with Bank at SUB 1 Online](image)
Figure 4. System Frequency Response with Bank at SUB 2 Online

Figure 5. System Frequency Response with Both Banks Online
for each harmonic were averaged over the entire sample time. Figure 6 shows the current and acoustic spectrums for SUB 1. The figure shows that the highest acoustic magnitudes are seen at the 2\textsuperscript{nd}, 14\textsuperscript{th}, 22\textsuperscript{nd}, and 24\textsuperscript{th} harmonics. Acoustic measurements were also taken at SUB 2, but simultaneous current data was not provided. Figure 7 shows the acoustic spectrum. The largest magnitudes of acoustic emission occur at the 2\textsuperscript{nd}, 4\textsuperscript{th}, 6\textsuperscript{th}, and 12\textsuperscript{th} harmonics.

![Figure 6. SUB 1 Current and Acoustic Spectrum](image)

In conducting literature research on the topic, Haskew found a source of relevant information in a study conducted by Cox and Guan from Louisiana Tech University [5]. The authors conducted a very similar study, determining why audible noise was created by a capacitor bank at a substation supplying power for an electric arc furnace. The paper briefly discusses the electric forces created by a simple parallel plate capacitor. Acoustic measurements of the noise created by a single capacitor bank when subjected to current harmonics are documented and compared to acoustic noise created by a 20 MVA transformer exposed to the same current harmonics. Impact testing of several capacitor banks was also conducted by the
authors and the results are given. The authors proposed that when a capacitor bank is exposed to current harmonics, electric forces created within the bank can excite the bank’s mechanical natural frequencies and create considerable audible noise. The authors state that “in rare instances (e.g., a substation located in a quiet neighborhood), the capacitor noise might be sufficiently objectionable such that it would be necessary to shield the capacitor bank or change the bank configuration to reduce the audible noise.” The authors also noted, through the results of the impact hammer testing, capacitor banks have many natural frequencies between 0 Hz and 2 kHz. After conducting vibration measurements at the substation while the bank was subjected to current harmonics, the authors found that “the vibration data obtained in the substation compared favorably with that predicted by transfer functions obtained from impact hammer tests in the laboratory.”

Mechanical natural frequencies being excited by current/voltage harmonics explains some of the oddities seen in Figure 6. At SUB 1, the two frequencies of largest acoustic
generation are the 22\textsuperscript{nd} and 24\textsuperscript{th} harmonic, but the harmonic current near these frequencies is less than the current near other frequencies. Large gains in the response of the system caused by resonant frequencies near these harmonics would explain this. Therefore, Haskew concluded that some vibration analysis was needed to determine the structural natural frequencies of a shunt capacitor.

Cox and Guan also made another important finding in determining that the audible noise created is a function of the current/voltage frequency mixed with the fundamental frequency. For example, their studies showed that “the 25\textsuperscript{th} harmonic voltage component (1,500 Hz) mixes with the fundamental component (60 Hz) to produce acoustic frequencies of 1,560 Hz and 1,440 Hz. Part of the acoustic energy at 1,440 Hz is also produced by the 23\textsuperscript{rd} harmonic and the fundamental mixing together.” This fact was also noticed by Haskew during his tests. As seen in Figure 6, considerable acoustic noise was measured at 1,320 Hz and 1,440 Hz, but the current measured at these frequencies was relatively low when compared to the current at 1,380 Hz. However, at 1,380 Hz, the magnitude of the acoustic noise measured was small. Applying this observation to Figure 7, harmonic excitation at the 3\textsuperscript{rd}, 5\textsuperscript{th}, 7\textsuperscript{th}, 11\textsuperscript{th}, and 13\textsuperscript{th} harmonic is inferred. Cox and Guan conclude the paper by stating that given more time, an electromagnetic shaker would be used to excite the bank instead of an impact hammer during the vibration testing.

\textit{Conclusions}

Audible noise is generated by capacitor banks at two substations that feed a DC electric arc furnace at a local steel mill. When the furnace is operating, the waveform of the capacitor current becomes distorted because of the non-linear operation of the mill’s 12 pulse rectifier. The FFT shows current spikes at the odd harmonics of 60 Hz, with dominant peaks occurring at the 5\textsuperscript{th}, 7\textsuperscript{th}, 11\textsuperscript{th}, 13\textsuperscript{th}, 23\textsuperscript{rd}, and 25\textsuperscript{th} harmonics. EMTP simulations of the system showed that there
are system electrical resonances near some of the frequencies of current injection, but the magnitude of the current at these frequencies is still only a few percent of the RMS current.

Acoustic measurements were taken at both substations. At SUB 1, the dominant frequencies of acoustic generation were the 2\textsuperscript{nd}, 14\textsuperscript{th}, 22\textsuperscript{nd}, and 24\textsuperscript{th} harmonics of 60 Hz, with the 22\textsuperscript{nd} and 24\textsuperscript{th} harmonic magnitude being the largest. At SUB 2, the dominant frequencies of acoustic generation were the 2\textsuperscript{nd}, 4\textsuperscript{th}, 6\textsuperscript{th}, and 12\textsuperscript{th} harmonics of 60 Hz, with the 2\textsuperscript{nd} and 4\textsuperscript{th} harmonic magnitude being the largest. While comparing the acoustic data to simultaneous current data, it was noticed that the frequencies with large acoustic magnitudes were at 60 Hz offsets of the frequencies with large current magnitudes.

Through literature research, another occurrence of this phenomenon was found in a paper by Cox and Guan. The authors conducted a very similar study, investigating why audible noise was created by a capacitor bank at a substation supplying power for an electric arc furnace. The authors propose that when a capacitor bank is exposed to current harmonics, electric forces created within the bank can excite mechanical natural frequencies of the system and create considerable audible noise. Cox and Guan determined that the audible noise created by the capacitors is a function of the current/voltage frequency mixed with the fundamental frequency. Therefore, the frequencies of acoustic generation are at 60 Hz offsets of the excitation frequency.

\textit{Goals of this Study}

The first goal of the study was determining how the capacitors are being excited. For this study, two 16.6 kV, 625 kVAR shunt capacitors were provided by the local utility. These capacitors are similar to those used at SUB 1 and SUB 2. One of the capacitors provided had failed in service and permission was given to dissect it. Through the capacitor dissection, information regarding the shell construction, capacitive element construction and orientation, and
construction material was to be determined. Using this information, estimates for the electric forces generated within these elements were to be made. Also, the root cause of the acoustic frequencies being at 60 Hz offsets of the current frequencies was to be determined.

After reading the study by Cox and Guan, it was thought that the excitation of mechanical resonances was causing the magnitude of the acoustic generation to be larger at some frequencies than others. The second goal of the study was to estimate, using theoretical methods, the natural frequencies of the capacitor bank. Methods used to calculate the capacitor natural frequencies were to be found through literature research. Also, the corresponding mode shapes at these natural frequencies were to be determined.

The third goal of the study was to verify the calculated natural frequencies with vibration analysis. The two shunt capacitors provided by the local utility provider were to be used as test specimens. Through the vibration analysis, the gains at the natural frequencies were to be determined. With the test equipment that was made available, the mode shapes were to also be experimentally determined.

Lastly, methods to reduce the magnitude of the audible noise were to be found. Through the EMTP-RV simulations, Haskew showed that filtering the current harmonics would be difficult because of the sporadic load drawn by the furnace and the changes in the system electrical resonances caused by switching the banks. His simulations are briefly explained. Using the natural frequencies, gains, and mode shapes determined through theoretical and experimental methods, methods to reduce the audible noise were to be determined.
CHAPTER 3
CAPACITOR DISSECTION AND ANALYSIS

For this study, the utility provider offered the use of two 16.6 kV, 625 kVAR shunt capacitors as test specimens. The capacitors are similar to those used at SUB 1 and SUB 2. One of the capacitors had failed in service and permission was given to dissect it. Before the capacitor was cut open, other tests were also performed. These tests are discussed in Chapter 5.

There were several reasons why the capacitor needed to be dissected. The first was the shell construction was unknown. Because vibration models were to be created, the shell thickness was needed. Also, it was unclear whether the capacitor had an internal support structure, which would have a large impact on the natural frequency of the bank. It was also unknown how the internal capacitive elements were arranged. As will be explained, the arrangement of the internal elements affects which sides of the bank are excited. Finally, the capacitor element properties were needed for electrostatic force calculations.

Before the capacitor was dissected, a literature review was performed to determine the typical construction of a shunt capacitor. The information obtained in this review is covered in the first section of this chapter. This includes typical methods and materials used to construct the capacitive elements and the internal arrangement of these elements. Also discussed are the layers of the capacitive elements and their properties as well as the forces generated within these elements.
The last section of this chapter discusses the dissection of the failed capacitor and subsequent analysis. The internal element construction, element arrangement, and material layers are discussed. Using the information gathered during the literature review, estimates for the internal forces generated are made. Lastly, the findings during this phase of the project are summarized.

*Literature Review*

During the initial phases of the project, little was known about the internal construction of the supplied shunt capacitors. It was thought that the capacitive elements were constructed of alternating layers of foil and dielectric, with the entire structure surrounded by a volume of liquid dielectric. However, the arrangement of the internal elements was uncertain. Before the capacitor was dissected, a literature search on shunt capacitors was performed first. The main goals of the literature research were to determine the internal construction of shunt capacitors, determine the typical capacitor case construction, and investigate electrostatic force generation inside capacitors.

Shunt capacitor banks are constructed using several capacitive elements connected in series/parallel configurations according to the desired capacitance [9]. To construct a capacitive element, a sheet of alternating layers of conducting material and dielectric material is first made. High voltage capacitors usually consist of aluminum foils interleaved with Kraft paper or polypropylene film. Depending on the capacitance needed, the length and width of the sheet are varied. The sheet is then tightly wound into either a rectangular or cylindrical shape. After connecting several elements in the appropriate configuration, the package is put in a mild steel or stainless steel container fitted with a lid. The capacitors inside the containers are dried under vacuum and the containers are then impregnated with an insulating fluid [12]. The construction
of a typical shunt capacitor is shown in Figure 8. This capacitor bank has capacitive elements wound in a rectangular shape and stacked vertically. Other configurations include spirally wound and stacked horizontally and rectangular wound and stacked horizontally.

![Figure 8. Typical Shunt Capacitor Construction with Elements Vertically Arranged [9]](image)

Regarding the forces generated by capacitors, the most useful resource found was a study by McDuff on power losses due to acoustic generation in capacitors [13]. Some information was also found in Cox and Guan’s study, but this also seems to be based on McDuff’s work. To determine the forces on a capacitor, McDuff considers a simple parallel plate capacitor in which one plate is free to move, similar to Figure 9. The capacitor charge is given by:

\[ q = C(y)v \]  

where

- \( q \) is the charge,
- \( y \) is the plate separation
- \( C(y) \) is the position dependent capacitance, and
- \( v \) is the time dependent voltage difference.
Knowing current is the time derivative of charge, McDuff uses this relationship along with basic differentiation rules to determine the current, given as:

\[ i = \frac{dq}{dt} = \frac{d}{dt} \left[ \int (y)v^-- \right] = C(y) \frac{dv}{dt} + v \frac{dC(y)}{dy} \frac{dy}{dt} \]  

(2)

where

\( i \) is the capacitor current.

Continuing along these lines, the power delivered to a capacitor is determined and given as:

\[ p = vi = v \frac{d}{dt} \left[ \int (y)v^- \right] = \frac{1}{2} \frac{d}{dt} \left[ \int (y)v^2^- \right] + \frac{1}{2} v^2 \frac{dC(y)}{dy} \frac{dy}{dt} \]  

(3)

where

\( p \) is the power delivered to the capacitor.

\[ p = \frac{dW}{dt} + f(y) \frac{dy}{dt} \]  

(4)

where

\( W \) is the capacitor electrical energy storage, and

Figure 9. Parallel Plate Capacitor with Variable Plate Separation [13]
$f(y)$ is the mechanical force generated.

From Equation 3 and Equation 4, the force acting on the capacitor is:

$$f(y) = \frac{1}{2} v^2 \frac{dC(y)}{dy}$$

(5)

McDuff notes that when the capacitor is charged, the forces act to decrease the plate separation. Figure 10 shows how the electric fields act between two charged plates. As seen, the fields outside the plates cancel each other out while the fields between the plates add. The plate with positive charge is pulled towards the negatively charged plate. In the case of an AC source, the capacitor only acts to reduce the separation distance in one direction. This is because the force generated by the capacitor is a function of the voltage squared. Unlike the voltage waveform, the voltage squared waveform does not change polarity, as shown in Figure 11. Therefore, the forces only act to contract the plate separation. This will be discussed further in near the end of the chapter.

Figure 10. Forces Generated Between Two Charged Plates
Figure 11. Voltage and Voltage Squared Waveform Comparison

McDuff also gives the capacitance of a wound capacitor element like those previously described. He begins by considering the element in the unwound state. This is shown in Figure 12. The capacitance of this layer is given as:

\[ C(y) = \frac{\varepsilon_0 \varepsilon_1 \varepsilon_2 A}{d\varepsilon_2 + 2\varepsilon_1 \left( \frac{y - D}{2} \right)} \]  

(6)

where

- \( \varepsilon_0 \) is the dielectric constant of vacuum,
- \( \varepsilon_1 \) is the dielectric film relative dielectric constant,
- \( \varepsilon_2 \) is the dielectric fluid relative dielectric constant,
- \( A \) is the area of the conducting plates, and
- \( D \) is the dielectric film thickness.
McDuff gives typical thickness values for each layer along with typical dielectric constants of the materials. These values are given in Table 1. Though McDuff calculates the capacitance of a wound capacitor with multiple dielectric layers, he does not use this capacitance for any force calculations. He instead further simplifies the model by using a single dielectric layer between the plates and assumes a constant voltage. This results in the force equation given as:

\[
f(y) = \frac{1}{2} \frac{AV^2}{y^2}
\]  

(7)

Applying the information given above to the rectangular wound, vertically stacked capacitor bank in Figure 8, it is concluded that the main excitation occurs on the narrow and broad sides of the container, with fringe fields contributing minimal excitation to the top and bottom of the container.

<table>
<thead>
<tr>
<th>Element Layers Measured by McDuff</th>
</tr>
</thead>
<tbody>
<tr>
<td>Layer</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>Aluminum Foil</td>
</tr>
<tr>
<td>Polypropylene</td>
</tr>
<tr>
<td>Silicon Oil</td>
</tr>
</tbody>
</table>
container. This is shown in Figure 13. The narrow sides see the largest excitation because the area of the capacitive element is largest on these sides. The excitation is caused by either the capacitive element directly contacting the container as it is relaxed or a pressure wave being produce in the dielectric fluid as the element is relaxed.

**Dissection**

As discussed earlier, a failed capacitor was donated by the utility provider for use as a test specimen. Information found in the literature review lacked complete details of typical capacitor construction, so dissection of the capacitor was essential to learn this specific information. Detailed characteristics of the capacitor construction were needed in order to perform subsequent analysis.

With the help of personnel at The University of Alabama College of Engineering (UACOE) machine shop, the capacitor was first drained of the liquid dielectric. From the capacitor nameplate, the dielectric fluid used was Edisol and total volume of the fluid was 3.23 gallons. Vibration analysis of the empty capacitor was then performed. This is discussed in Chapter 5. After testing was completed, the capacitor was sent back to the UACOE machine shop. It was desired to test the shell after the internal components were removed so one of the
broad sides of the shell was removed. Removing one of the broad sides was thought to have the least effect on the structure while also allowing the internal structure to be easily removed. The broad side was cut using a pneumatic cut off tool while cooling the cutting edge with water. Figure 14 shows the capacitor with the broad side removed. As seen in Figure 14, the entire internal structure is covered with a layer of Kraft paper. Figure 15 shows the internal structure with the Kraft paper peeled back. There were 16 capacitive elements stacked in a vertical configuration. The failure point was observed immediately and is shown in Figure 16.

Figure 14. Failed Capacitor with Broad-Side Plate Removed
Figure 15. Failed Capacitor with Kraft Paper Peeled Back

Figure 16. Observed Point of Failure
After cutting the shell, the internal elements of the capacitor were removed. The exposed Kraft paper was cut out and the leads were also cut. No internal structural support was used to hold the elements in place. However, the elements were packed tightly and could not freely move. After loosening one of the elements and removing it, the remaining elements were easily taken out. Figure 17 shows the removed internal structure.

![Figure 17. Internal Structure of the Failed Capacitor](image)

One of the capacitive elements was dissected next. This element is shown in Figure 18. Each capacitive element was constructed by rolling a sheet of layered polypropylene (the dielectric) and aluminum foil. The element was approximately 0.825 inches thick, 5 3/4 inches wide, and 19 5/8 inches long. There were three sheets of polypropylene and two sheets of aluminum per layer. The volume between sheets was impregnated with the Edisol dielectric. The layers of the element are shown in Figure 19. As shown, the first two sheets are polypropylene, followed by a sheet of aluminum foil, another sheet of polypropylene, and a final sheet of aluminum foil. The sheet of polypropylene and aluminum had a thickness of approximately
0.004 inches (4 mils). The individual aluminum layers had a thickness between 0.0004 inches (0.4 mils) and 0.0008 inches (0.8 mils) and the polypropylene layers had a thickness between 0.0008 inches (0.8 mils) and 0.0012 inches (1.2 mils). These measurements are approximately equal to those shown in Table 1.

![Figure 18. Capacitive Element with Recorded Dimensions](image)

The element connections were examined next to determine the nominal capacitance of each element. Figure 20 shows how the elements are wired. From Figure 20, it was determined
that the 16 elements are divided into pairs of two and wired in parallel. These 8 pairs are then wired in series. This equivalent circuit is shown in Figure 21. Knowing that the nominal capacitance of all the elements is 6 \( \mu F \) and applying circuit theory, it was determined that the nominal capacitance of each element is 24 \( \mu F \). From the nominal capacitance, it is estimated that the unwound layer is approximately 240 feet long.

![Figure 20. Observed Element Connections](image)

![Figure 21. Equivalent Element Connections](image)
Analysis

After dissecting the capacitor, the information found during the literature review was applied to the test specimen. First, the forces generated by the capacitive elements were determined. Differentiating Equation 6 with respect to $y$ yields the equation for position dependent capacitance, given by:

$$\frac{dC(y)}{dy} = -\frac{\varepsilon_0\varepsilon_1^2\varepsilon_2 A}{D\varepsilon_2 + 2\varepsilon_1\left(\frac{y-D}{2}\right)^2}$$

(8)

Inserting Equation 8 into Equation 5 yields the force generated by the wound capacitive element in the relaxed state given by:

$$f(y) = -\frac{\varepsilon_0\varepsilon_1^2\varepsilon_2 A y^2}{2\left[D\varepsilon_2 + 2\varepsilon_1\left(\frac{y-D}{2}\right)^2\right]^2}$$

(9)

The previous equations apply to the layer shown in Figure 19. However, this relaxed layer is wound to create the element. Figure 22 gives an example of the layers of Figure 19 wound. As shown, there is not only an attraction within each layer but there is also an attraction between the layers. If the dielectric thickness within the layer was the same as between layers, the forces would cancel. However, there are two layers of solid dielectric and three layers of liquid dielectric between the plates within different layers instead of one layer of solid dielectric and two layers of liquid dielectric within the same layer. This means the capacitance between layers is less than the capacitance within a layer. Therefore, the forces do not cancel within the internal layers. The capacitance between layers is calculated by Equation 10, given as:
\[ C(y) = \frac{\varepsilon_0 \varepsilon_1 \varepsilon_2 A}{2D\varepsilon_2 + 3\varepsilon_1 \left( \frac{y-2D}{3} \right)} \quad (10) \]

Differentiating Equation 10 with respect to \( y \) yields the equation for position dependent capacitance between the layers, given by:

\[ \frac{dC(y)}{dy} = -\frac{\varepsilon_0 \varepsilon_1^2 \varepsilon_2 A}{\left[ 2D\varepsilon_2 + 3\varepsilon_1 \left( \frac{y-2D}{3} \right) \right]^2} \quad (11) \]

Inserting Equation 11 into Equation 5 yields the force generated between two layers as:

\[ f(y) = -\frac{\varepsilon_0 \varepsilon_1^2 \varepsilon_2 A v^2}{2 \left[ 2D\varepsilon_2 + 3\varepsilon_1 \left( \frac{y-2D}{3} \right) \right]^2} \quad (12) \]

Combining Equation 9 and Equation 12 yields the total force acting on the positively charged plates. Assuming the area of the plates in different layers is approximately the same, this force can be calculated using:

\[ f_{\text{total}}(y) = \varepsilon_0 \varepsilon_1^2 \varepsilon_2 A v^2 \times \left[ -\frac{1}{2 \left[ D\varepsilon_2 + 2\varepsilon_1 \left( \frac{y-D}{2} \right) \right]^2} + \frac{1}{2 \left[ 2D\varepsilon_2 + 3\varepsilon_1 \left( \frac{y-2D}{3} \right) \right]^2} \right] \quad (13) \]
Figure 22. Direction of Forces for Wound Section

Equation 9 applies to the interior layers of the element and Equation 13 describes the forces in the exterior layer of each element. Therefore, the largest forces generated occur in the exterior layer of each element, with smaller forces generated internally. All forces act to compress the element.
For the outer layer, assuming the force is uniformly distributed, the pressure generated is found by calculating the force per unit area, given as:

\[ p_r = \frac{f(y)}{A} = -\frac{\varepsilon_0 \varepsilon_1^2 \varepsilon_2 v^2}{2 \left[ D \varepsilon_2 + 2 \varepsilon_1 \left( \frac{y - D}{2} \right) \right]^2} \]  

where

\( p_r \) is the generated pressure.

Next, the limits of the pressure were determined. To estimate the largest possible pressure generated, let the dielectric thickness go to zero (set \( y \) equal to \( D \)). This is shown in Equation 15, given by:

\[ p_{\text{max}} = -\frac{\varepsilon_0 \varepsilon_1^2 \varepsilon_2 v^2}{2 \left[ \varepsilon_2 \right]^3} \]  

where

\( p_{\text{max}} \) is the maximum amount of pressure generation possible.

The minimum amount of pressure generated when the voltage peaks is found by setting \( y \) equal to the relaxed measurement of the dielectric in Equation 9. As an example, if the voltage difference over the entire bank is 16.6 kV (2,075 V per element) and the layer properties in Table 1 are used, then the maximum and minimum pressures are shown in Figure 23. Applying the measurements recorded for the outer layer, the limit for the maximum force generated on the
Figure 23. Calculated Maximum and Minimum Pressure Generation: Broad Side

broad sides of the element is approximately 900 lbf. For the narrow sides of the element, the limit for the maximum force generated is approximately 130 lbf. In the relaxed state, the limit for the minimum force generated on the broad side is approximately 50 lbf. For the narrow side, the limit for the minimum force generated is approximately 7 lbf.

Non-Linear Excitation

As discussed in Chapter 2, Haskew collected current and acoustic field data at the two substations generating audible noise [2]. After the data was analyzed, it was noticed that frequencies that had large current magnitudes had low acoustic levels. It was also noticed that high acoustic levels were seen at 60 Hz offsets of the current frequencies. This can be seen in Figure 6. For example, a large current magnitude is seen at the 23rd harmonic, but the acoustic magnitude is low. However, at the 22nd and 24th harmonic, the acoustic magnitude is high while the current magnitude is small. This was also noted by Cox and Guan during their study [1]. They noted no current at the frequencies with acoustic levels. Cox and Guan concluded during
their study that the harmonic voltage components were mixing with the fundamental component (60 Hz component) to produce sidebands of 60 Hz. These sidebands indicate non-linear behavior is occurring.

From Equation 5, it is shown that the forces generated by the internal elements are a function of the voltage squared. It was also shown in Figure 11 that squaring a sinusoidal voltage waveform would double the frequency. Therefore, the current/voltage harmonics that the capacitor bank sees would excite the bank at twice the frequency. This explains why Haskew’s acoustic data showed acoustic generation at even harmonics while the excitation was at odd harmonics. If you double an odd harmonic frequency, you get an even harmonic frequency.

The previous thoughts were tested using the current data collected at SUB 1. Using the rated reactive power capacity at the rated voltage, it was determined that the equivalent capacitance of the bank at SUB 1 is approximately 3 µF. Because the capacitance and current was known, the voltage can be calculated with the relationship:

\[ v = \frac{1}{C} \int_0^t i(\tau) d\tau \]  

where

\( C \) is the total capacitance of the bank, 3µF

The discrete current data was integrated for each phase using the trapezoidal rule. Figure 24 shows the capacitor bank voltage for each phase. Figure 25 shows the FFT of the voltage squared. The FFT of phase b and c show peaks only at the even harmonics, as was expected. Phase a shows only even peak until the 19th harmonic, at which point the peaks switch back to the odd harmonics. However, the integration is not perfect. The initial conditions for phases “a” and “c” are estimates. Phase “b” is the most accurate as the initial condition here is known (0
volts). In reality, all of the phases should only have even harmonics in the voltage squared waveform because, as shown in Figure 2, all of the phases have the same odd harmonics present in the current waveform.

Figure 24. Estimated Voltage at SUB 1 Bank
For this study, the local utility provider provided a failed capacitor that was dissected to learn details of shunt capacitor construction. This capacitor is typical of those used at both of the substation banks generating audible noise. From the dissection of the capacitor and a literature review, it was determined that the capacitor is made of several rectangular elements that are wound from layers of aluminum foil and polypropylene. The elements are wired in a series/parallel configuration and placed vertically inside of a shell made of 1/16 inch stainless steel. Because the area of the plate parallel to the narrow side of the shell is the largest, the main forces are generated on the narrow side of the capacitor. However, considerable forces are also generated on the broad side of the capacitor as well. The forces generated act to constrict the

Figure 25. FFT of the Voltage Squared Waveform

Conclusions

For this study, the local utility provider provided a failed capacitor that was dissected to learn details of shunt capacitor construction. This capacitor is typical of those used at both of the substation banks generating audible noise. From the dissection of the capacitor and a literature review, it was determined that the capacitor is made of several rectangular elements that are wound from layers of aluminum foil and polypropylene. The elements are wired in a series/parallel configuration and placed vertically inside of a shell made of 1/16 inch stainless steel. Because the area of the plate parallel to the narrow side of the shell is the largest, the main forces are generated on the narrow side of the capacitor. However, considerable forces are also generated on the broad side of the capacitor as well. The forces generated act to constrict the
rectangular elements. The shell of the capacitor is either excited by the capacitive element directly contacting the container as it is relaxed or a pressure wave being produce in the dielectric fluid as the element is relaxed. Using measurements made of the capacitive layers, the compressive pressure generated within the rectangular elements was estimated and the associated forces are considerable.

From the acoustic data gathered at SUB 1, it was noticed that the frequencies of acoustic generation occur at 60 Hz offsets of the current harmonics. It was determined that this is a result of the electrical forces generated being a function of the voltage squared. As a result, current at even harmonics creates excitation at odd harmonics. This was validated using current data gathered at SUB 1.
CHAPTER 4

SHELL MODELING

An important facet of this study was determining how the electrical forces created by the capacitive elements cause mechanical vibrations that are the source of the acoustic problem. After seeing the internal structure of the capacitor, it is apparent that, from a structural dynamics viewpoint, the structure is complex. The difficulty comes not only from the dynamics of the shell but also the large internal mass of the capacitive elements and the coupling from the dielectric fluid. Though the entire structure was not modeled, the shell of the capacitor was modeled using simple finite element analysis and the results are discussed. Experimental data was taken to validate the analysis and is presented in Chapter 5.

Blevins’s Methods

For introductory analysis, the natural frequencies and mode shapes of a plate were investigated. This may seem somewhat simplistic, but with the correct boundary conditions, the sides of the capacitor shell can be seen as plates. There are several sources available on analyzing plates, but the most useful and simple source found was in Blevins’s work [4]. Blevins simplifies the calculation of natural frequencies of plates by providing easy to follow calculations and tables of coefficients needed for a variety of boundary conditions.

The first step of the analysis was determining the appropriate boundary conditions. The sides of the capacitor are created by bending a plate into a hollow rectangle and fastening it with
a weld. Then a top and bottom are welded to the bent plate to complete the shell. From the construction, it was decided that the correct boundary conditions lie somewhere between simply supported and clamped edges, or a combination of the two. Simple support conditions at the edges set the displacement of the plate edge to zero, but the slope at the edge of the plate can change (the sides do not necessarily have to be perpendicular), as seen in Figure 26. A clamped support is similar to the simple support in that the displacement conditions at the ends are zero; however the slope at the edges must also be zero. This is also illustrated in Figure 26. Both conditions are exaggerated for clarity. The natural frequencies of a plate simply supported along all four edges were determined using Equation 16 from Table 11.4 of Blevins, given as:

\[
 f_{ij} = \frac{\pi^2 \times [i^2 + j^2 (\frac{a}{b})^2]}{2\pi \times a^2} \times \left[ \frac{Eh^3}{12\gamma(1-\nu^2)} \right]^{1/2}
\]

where

- \(f_{ij}\) is the natural frequency of each mode,
- \(i,j\) are the number of half waves in a mode shape along the horizontal/vertical axis respectively,
- \(a\) is the length of the plate,
$b$ is the width of the plate,

$E$ is the modulus of elasticity of the plate material,

$h$ is the thickness of the plate,

$\gamma$ is the mass per unit area of the plate (density ($\rho$) times thickness ($h$)), and

$\nu$ is Poisson’s ratio of the plate material.

For the entire shell, the properties for stainless steel were used ($E = 28 \times 10^6$ psi, $\nu = 0.3$, $\rho = 0.29$ lb/in$^3$), and the thickness of the plates is 1/16 inches [6]. For the broadsides, the length of the plate is 22 1/2 inches, and the width of the plate is 13 1/2 inches. The narrow sides have a plate length of 22 1/2 inches and a plate width of 6 inches. Using these parameters, the natural frequencies from 0 Hz to 3 kHz were found for both the broad side and the narrow side and are shown in Table 2 and Table 3. From Equation 17, the natural frequency is mainly a function of the plate thickness, $h$, and the width, $a$. Because both plates are very thin (1/16 inches), there are many natural frequencies present. The broad side also has many more natural frequencies than the narrow side because the plate width is much larger. For a simply supported plate, the mode shapes can be found using Equation 11-22 of Blevins, given as:

$$
\tilde{Z}_q(x, y) = \sin\left(\frac{i \pi x}{a}\right) \times \sin\left(\frac{j \pi y}{b}\right)
$$

where

$Z$ is mode shape,

$x$ is the distance along the width of the plate, and

$y$ is the distance along the length of the plate.
As an example, the broad side mode shapes for 685 Hz (i=4, j=4) and 912 Hz (i=4, j=6) are shown in Figure 27. As shown, the mode shapes for a plate simply supported along all four edges are made up of a several alternating half waves, similar to a checker board pattern. The amplitude is arbitrary and has been normalized to one. For a plate with clamped conditions on all sides, the natural frequencies are approximated using Equation 11-21 from Blevins, given as:

\[
f_{ij} \approx \frac{\pi}{2} \left[ \frac{G_1^4}{a^4} + \frac{G_2^4}{b^4} + \frac{2J_1J_2}{a^2b^2} \right]^{1/2} \left[ \frac{Eh^3}{12\gamma(1-\nu^2)} \right]^{1/2}
\]

where

- \( G_1 \) is a dimensionless parameter \( i + 1/2 \),
- \( G_2 \) is a dimensionless parameter \( j + 1/2 \),
- \( J_1 \) is the dimensionless parameter \( (i + 1/2)^2 \times \left[ 1 - \frac{2}{(i+1/2)\pi} \right] \),
- \( J_2 \) is the dimensionless parameter \( (j + 1/2)^2 \times \left[ 1 - \frac{2}{(j+1/2)\pi} \right] \).

Using the same dimensions and properties specified for the simply supported plate, the natural frequencies from 0 Hz to 3 kHz for the clamped plates were calculated and are given in Table 4 and Table 5. Comparing the end conditions, it appears that the plate with clamped
conditions has higher natural frequencies than the plate with simple conditions. For a clamped plate, the mode shapes can be approximated using the more general form of Equation 11-22 of Blevins, given as:

\[
\tilde{Z}_j(x, y) \approx \tilde{X}_i(x) \tilde{Y}_j(y)
\]

where

\( X \) is mode shape in the x direction, and

\( Y \) is the mode shape in the y direction.

Blevins notes that, for plates without free ends, the mode shapes are generally made up of half waves. This is shown in Figure 28, which gives the mode shapes of clamped and simple supported beams. As shown, the mode shapes of both are similar, except for the displacement of the plates at the ends. This means that the fully clamped plate will have also have mode shapes with a checkerboard pattern, but the curvature of the shapes will be slightly different.

Figure 28. Mode Shapes for a Beam with Simply Supported and Clamped Ends [4]
### Table 2

Calculated Natural Frequencies for Broad Side with Simple Conditions (Hz)

<table>
<thead>
<tr>
<th>Number of Half Waves in X</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>43</td>
<td>137</td>
<td>295</td>
<td>515</td>
<td>1,145</td>
<td>1,554</td>
<td>2,026</td>
<td>2,562</td>
<td>3,160</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>77</td>
<td>171</td>
<td>329</td>
<td>549</td>
<td>1,179</td>
<td>1,588</td>
<td>2,060</td>
<td>2,596</td>
<td>3,194</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>134</td>
<td>228</td>
<td>385</td>
<td>606</td>
<td>1,236</td>
<td>1,645</td>
<td>2,117</td>
<td>2,652</td>
<td>3,251</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>213</td>
<td>307</td>
<td>465</td>
<td>685</td>
<td>1,315</td>
<td>1,724</td>
<td>2,196</td>
<td>2,732</td>
<td>3,330</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>315</td>
<td>409</td>
<td>567</td>
<td>787</td>
<td>1,071</td>
<td>1,417</td>
<td>1,826</td>
<td>2,298</td>
<td>2,834</td>
<td>3,432</td>
</tr>
<tr>
<td>6</td>
<td>440</td>
<td>534</td>
<td>691</td>
<td>912</td>
<td>1,195</td>
<td>1,542</td>
<td>1,951</td>
<td>2,423</td>
<td>2,958</td>
<td>3,557</td>
</tr>
<tr>
<td>7</td>
<td>587</td>
<td>681</td>
<td>839</td>
<td>1,059</td>
<td>1,343</td>
<td>1,689</td>
<td>2,098</td>
<td>2,571</td>
<td>3,106</td>
<td>3,704</td>
</tr>
<tr>
<td>8</td>
<td>757</td>
<td>851</td>
<td>1,009</td>
<td>1,229</td>
<td>1,513</td>
<td>1,859</td>
<td>2,268</td>
<td>2,741</td>
<td>3,276</td>
<td>3,874</td>
</tr>
<tr>
<td>9</td>
<td>950</td>
<td>1,044</td>
<td>1,202</td>
<td>1,422</td>
<td>1,705</td>
<td>2,052</td>
<td>2,461</td>
<td>2,933</td>
<td>3,469</td>
<td>4,067</td>
</tr>
<tr>
<td>10</td>
<td>1,165</td>
<td>1,259</td>
<td>1,417</td>
<td>1,637</td>
<td>1,921</td>
<td>2,267</td>
<td>2,676</td>
<td>3,149</td>
<td>3,684</td>
<td>4,282</td>
</tr>
<tr>
<td>11</td>
<td>1,403</td>
<td>1,497</td>
<td>1,655</td>
<td>1,875</td>
<td>2,159</td>
<td>2,505</td>
<td>2,914</td>
<td>3,387</td>
<td>3,922</td>
<td>4,520</td>
</tr>
<tr>
<td>12</td>
<td>1,664</td>
<td>1,758</td>
<td>1,916</td>
<td>2,136</td>
<td>2,419</td>
<td>2,766</td>
<td>3,175</td>
<td>3,647</td>
<td>4,183</td>
<td>4,781</td>
</tr>
<tr>
<td>13</td>
<td>1,947</td>
<td>2,042</td>
<td>2,199</td>
<td>2,419</td>
<td>2,703</td>
<td>3,049</td>
<td>3,458</td>
<td>3,931</td>
<td>4,466</td>
<td>5,064</td>
</tr>
<tr>
<td>14</td>
<td>2,253</td>
<td>2,348</td>
<td>2,505</td>
<td>2,725</td>
<td>3,009</td>
<td>3,355</td>
<td>3,764</td>
<td>4,237</td>
<td>4,772</td>
<td>5,370</td>
</tr>
<tr>
<td>15</td>
<td>2,582</td>
<td>2,676</td>
<td>2,834</td>
<td>3,054</td>
<td>3,338</td>
<td>3,684</td>
<td>4,093</td>
<td>4,565</td>
<td>5,101</td>
<td>5,699</td>
</tr>
<tr>
<td>16</td>
<td>2,933</td>
<td>3,028</td>
<td>3,185</td>
<td>3,406</td>
<td>3,689</td>
<td>4,035</td>
<td>4,445</td>
<td>4,917</td>
<td>5,452</td>
<td>6,050</td>
</tr>
</tbody>
</table>

### Table 3

Calculated Natural Frequencies for Narrow Side with Simple Conditions (Hz)

<table>
<thead>
<tr>
<th>Number of Half Waves in X</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>171</td>
<td>649</td>
<td>1,446</td>
<td>2,562</td>
</tr>
<tr>
<td>2</td>
<td>205</td>
<td>683</td>
<td>1,480</td>
<td>2,596</td>
</tr>
<tr>
<td>3</td>
<td>261</td>
<td>740</td>
<td>1,537</td>
<td>2,652</td>
</tr>
<tr>
<td>4</td>
<td>341</td>
<td>819</td>
<td>1,616</td>
<td>2,732</td>
</tr>
<tr>
<td>5</td>
<td>443</td>
<td>921</td>
<td>1,718</td>
<td>2,834</td>
</tr>
<tr>
<td>6</td>
<td>567</td>
<td>1,046</td>
<td>1,843</td>
<td>2,958</td>
</tr>
<tr>
<td>7</td>
<td>715</td>
<td>1,193</td>
<td>1,990</td>
<td>3,106</td>
</tr>
<tr>
<td>8</td>
<td>885</td>
<td>1,363</td>
<td>2,160</td>
<td>3,276</td>
</tr>
<tr>
<td>9</td>
<td>1,078</td>
<td>1,556</td>
<td>2,353</td>
<td>3,469</td>
</tr>
<tr>
<td>10</td>
<td>1,293</td>
<td>1,771</td>
<td>2,568</td>
<td>3,684</td>
</tr>
<tr>
<td>11</td>
<td>1,531</td>
<td>2,009</td>
<td>2,806</td>
<td>3,922</td>
</tr>
<tr>
<td>12</td>
<td>1,792</td>
<td>2,270</td>
<td>3,067</td>
<td>4,183</td>
</tr>
<tr>
<td>13</td>
<td>2,075</td>
<td>2,553</td>
<td>3,350</td>
<td>4,466</td>
</tr>
<tr>
<td>14</td>
<td>2,381</td>
<td>2,859</td>
<td>3,656</td>
<td>4,772</td>
</tr>
<tr>
<td>15</td>
<td>2,710</td>
<td>3,188</td>
<td>3,985</td>
<td>5,101</td>
</tr>
<tr>
<td>16</td>
<td>3,061</td>
<td>3,539</td>
<td>4,336</td>
<td>5,452</td>
</tr>
</tbody>
</table>

42
### Table 4
Calculated Natural Frequencies for Broad Side with Clamped Conditions (Hz)

<table>
<thead>
<tr>
<th>Number of Half Waves in X</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>83</td>
<td>209</td>
<td>398</td>
<td>651</td>
<td>966</td>
<td>1,344</td>
<td>1,785</td>
<td>2,289</td>
<td>2,855</td>
<td>3,485</td>
</tr>
<tr>
<td>2</td>
<td>120</td>
<td>243</td>
<td>433</td>
<td>685</td>
<td>1,001</td>
<td>1,379</td>
<td>1,820</td>
<td>2,324</td>
<td>2,891</td>
<td>3,521</td>
</tr>
<tr>
<td>3</td>
<td>183</td>
<td>302</td>
<td>490</td>
<td>742</td>
<td>1,057</td>
<td>1,436</td>
<td>1,877</td>
<td>2,382</td>
<td>2,949</td>
<td>3,579</td>
</tr>
<tr>
<td>4</td>
<td>272</td>
<td>386</td>
<td>571</td>
<td>822</td>
<td>1,137</td>
<td>1,515</td>
<td>1,957</td>
<td>2,461</td>
<td>3,029</td>
<td>3,659</td>
</tr>
<tr>
<td>5</td>
<td>384</td>
<td>495</td>
<td>677</td>
<td>925</td>
<td>1,239</td>
<td>1,617</td>
<td>2,058</td>
<td>2,563</td>
<td>3,131</td>
<td>3,761</td>
</tr>
<tr>
<td>6</td>
<td>519</td>
<td>628</td>
<td>807</td>
<td>1,053</td>
<td>1,365</td>
<td>1,742</td>
<td>2,183</td>
<td>2,688</td>
<td>3,255</td>
<td>3,885</td>
</tr>
<tr>
<td>7</td>
<td>678</td>
<td>785</td>
<td>961</td>
<td>1,205</td>
<td>1,515</td>
<td>1,891</td>
<td>2,331</td>
<td>2,835</td>
<td>3,402</td>
<td>4,033</td>
</tr>
<tr>
<td>8</td>
<td>859</td>
<td>965</td>
<td>1,139</td>
<td>1,381</td>
<td>1,689</td>
<td>2,064</td>
<td>2,503</td>
<td>3,006</td>
<td>3,572</td>
<td>4,203</td>
</tr>
<tr>
<td>9</td>
<td>1,063</td>
<td>1,168</td>
<td>1,341</td>
<td>1,580</td>
<td>1,887</td>
<td>2,260</td>
<td>2,698</td>
<td>3,200</td>
<td>3,766</td>
<td>4,396</td>
</tr>
<tr>
<td>10</td>
<td>1,289</td>
<td>1,395</td>
<td>1,566</td>
<td>1,804</td>
<td>2,109</td>
<td>2,480</td>
<td>2,916</td>
<td>3,417</td>
<td>3,983</td>
<td>4,612</td>
</tr>
<tr>
<td>11</td>
<td>1,539</td>
<td>1,644</td>
<td>1,814</td>
<td>2,050</td>
<td>2,354</td>
<td>2,723</td>
<td>3,158</td>
<td>3,658</td>
<td>4,223</td>
<td>4,851</td>
</tr>
<tr>
<td>12</td>
<td>1,811</td>
<td>1,915</td>
<td>2,085</td>
<td>2,320</td>
<td>2,622</td>
<td>2,990</td>
<td>3,424</td>
<td>3,923</td>
<td>4,486</td>
<td>5,144</td>
</tr>
<tr>
<td>13</td>
<td>2,105</td>
<td>2,210</td>
<td>2,379</td>
<td>2,613</td>
<td>2,914</td>
<td>3,280</td>
<td>3,713</td>
<td>4,210</td>
<td>4,773</td>
<td>5,400</td>
</tr>
<tr>
<td>14</td>
<td>2,423</td>
<td>2,527</td>
<td>2,696</td>
<td>2,929</td>
<td>3,229</td>
<td>3,594</td>
<td>4,025</td>
<td>4,522</td>
<td>5,083</td>
<td>5,709</td>
</tr>
<tr>
<td>15</td>
<td>2,763</td>
<td>2,867</td>
<td>3,035</td>
<td>3,268</td>
<td>3,567</td>
<td>3,931</td>
<td>4,361</td>
<td>4,856</td>
<td>5,416</td>
<td>6,041</td>
</tr>
<tr>
<td>16</td>
<td>3,126</td>
<td>3,230</td>
<td>3,398</td>
<td>3,630</td>
<td>3,928</td>
<td>4,291</td>
<td>4,720</td>
<td>5,214</td>
<td>5,773</td>
<td>6,397</td>
</tr>
</tbody>
</table>

### Table 5
Calculated Natural Frequencies for Narrow Side with Clamped Conditions (Hz)

<table>
<thead>
<tr>
<th>Number of Half Waves in X</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>368</td>
<td>1,007</td>
<td>1,965</td>
</tr>
<tr>
<td>2</td>
<td>394</td>
<td>1,037</td>
<td>1,997</td>
</tr>
<tr>
<td>3</td>
<td>441</td>
<td>1,086</td>
<td>2,048</td>
</tr>
<tr>
<td>4</td>
<td>513</td>
<td>1,157</td>
<td>2,120</td>
</tr>
<tr>
<td>5</td>
<td>609</td>
<td>1,249</td>
<td>2,213</td>
</tr>
<tr>
<td>6</td>
<td>732</td>
<td>1,365</td>
<td>2,329</td>
</tr>
<tr>
<td>7</td>
<td>881</td>
<td>1,505</td>
<td>2,466</td>
</tr>
<tr>
<td>8</td>
<td>1,055</td>
<td>1,670</td>
<td>2,628</td>
</tr>
<tr>
<td>9</td>
<td>1,253</td>
<td>1,859</td>
<td>2,812</td>
</tr>
<tr>
<td>10</td>
<td>1,475</td>
<td>2,073</td>
<td>3,021</td>
</tr>
<tr>
<td>11</td>
<td>1,721</td>
<td>2,311</td>
<td>3,253</td>
</tr>
<tr>
<td>12</td>
<td>1,990</td>
<td>2,574</td>
<td>3,510</td>
</tr>
<tr>
<td>13</td>
<td>2,282</td>
<td>2,861</td>
<td>3,791</td>
</tr>
<tr>
<td>14</td>
<td>2,598</td>
<td>3,171</td>
<td>4,095</td>
</tr>
<tr>
<td>15</td>
<td>2,936</td>
<td>3,505</td>
<td>4,424</td>
</tr>
<tr>
<td>16</td>
<td>3,298</td>
<td>3,863</td>
<td>4,776</td>
</tr>
</tbody>
</table>
The previous calculations made for the plate give the limits for the shell’s natural frequencies. In reality, the sides of the shell are a combination of both simple support conditions and clamped conditions. These combinations can be modeled using Blevins’s techniques, but this would require a great deal of calculation. Also, the mode shapes cannot be easily determined with mixed boundary conditions. Another method was needed in order to model more of the complexities of the system and reduce the calculation effort required. It was decided to use FEA software for the remaining analysis.

**FEA Analysis**

In Chapter 5, the experimental vibration analysis of a freely supported plate is discussed. The plate was tested in order to learn proper experimental test setup. Similarly, an FEA model of this plate was created first so that the software basics could be learned. The computed natural frequencies are shown in Table 6. The first twenty mode shapes, neglecting the six rigid body modes, are shown in Figure 29 and Figure 30.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency (Hz)</th>
<th>Convergence (%)</th>
<th>Mode</th>
<th>Frequency (Hz)</th>
<th>Convergence (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>396</td>
<td>0.2%</td>
<td>14</td>
<td>1,894</td>
<td>0.1%</td>
</tr>
<tr>
<td>8</td>
<td>530</td>
<td>0.1%</td>
<td>15</td>
<td>2,170</td>
<td>0.1%</td>
</tr>
<tr>
<td>9</td>
<td>811</td>
<td>0.1%</td>
<td>16</td>
<td>2,487</td>
<td>0.1%</td>
</tr>
<tr>
<td>10</td>
<td>965</td>
<td>0.1%</td>
<td>17</td>
<td>2,942</td>
<td>0.2%</td>
</tr>
<tr>
<td>11</td>
<td>1,097</td>
<td>0.2%</td>
<td>18</td>
<td>2,969</td>
<td>0.2%</td>
</tr>
<tr>
<td>12</td>
<td>1,540</td>
<td>0.1%</td>
<td>19</td>
<td>3,179</td>
<td>0.3%</td>
</tr>
<tr>
<td>13</td>
<td>1,876</td>
<td>0.2%</td>
<td>20</td>
<td>3,425</td>
<td>0.3%</td>
</tr>
</tbody>
</table>
Figure 29. Mode Shapes for a Free Plate: 396 Hz to 2,170 Hz
After validating the simulation results for the plate, the shell was modeled. As discussed in Chapter 4, one of the broad sides of the shell was cut off in order to remove the capacitive elements. The shell, minus the broad side, was then tested using the laser vibrometer to determine the natural frequencies. This shell was modeled in the FEA software and a modal analysis was performed. Figure 31 shows the shell that was drawn using the dimensions of the actual shell. The shell was created using 1/16 inch thick stainless steel. Surface meshing was used for creating the mesh. In total, six simulations were run with different frequency ranges. These ranges were 0 Hz to 500 Hz, 250 Hz to 750 Hz, 500 Hz to 1 kHz, 750 Hz to 1,250 Hz, 1 kHz to 1,500 Hz, and 1,250 Hz to 1,750 Hz. Staggering the frequency ranges this way provided
the best percent convergence and kept the simulations from failing. For all of the simulations, the solution was generated using a multi pass adaptive method with a maximum polynomial order of six and the convergence was set to 10%. Table 7 shows most of the calculated natural frequencies and the percent convergence. As shown, all of the calculated natural frequencies have a rate of convergence below 10%. Some of the natural frequencies do not fall within the bounds determined using Blevins’s methods. This is because one of edges of the narrow side is free. However, the mode shapes with motion on the broad sides do fall within these bounds. Figure 32 and Figure 33 show some of the mode shapes calculated at or near the harmonics of 60 Hz. Most of the mode shapes have elements of the checkerboard pattern that was created using the equations from Blevins along with simple support end conditions. However, there are some frequencies that have unexpected mode shapes, like 415 Hz and 551 Hz. This behavior is most likely caused by the free end condition of the unsupported sides, which was not modeled using Blevins’s method. The results from the FEA analysis show that, in this case, there are natural frequencies near most of the frequencies of acoustic emission.

![Figure 31. Shell Without Broad Side for FEA Analysis](image-url)
<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Convergence (%)</th>
<th>Frequency (Hz)</th>
<th>Convergence (%)</th>
<th>Frequency (Hz)</th>
<th>Convergence (%)</th>
<th>Frequency (Hz)</th>
<th>Convergence (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0%</td>
<td>403</td>
<td>2.7%</td>
<td>869</td>
<td>1.0%</td>
<td>1,242</td>
<td>2.0%</td>
</tr>
<tr>
<td>0</td>
<td>0.0%</td>
<td>410</td>
<td>2.3%</td>
<td>880</td>
<td>0.6%</td>
<td>1,248</td>
<td>4.2%</td>
</tr>
<tr>
<td>0</td>
<td>0.0%</td>
<td>415</td>
<td>3.1%</td>
<td>894</td>
<td>1.0%</td>
<td>1,269</td>
<td>4.9%</td>
</tr>
<tr>
<td>0</td>
<td>0.0%</td>
<td>439</td>
<td>2.4%</td>
<td>900</td>
<td>3.3%</td>
<td>1,268</td>
<td>1.0%</td>
</tr>
<tr>
<td>0</td>
<td>0.0%</td>
<td>442</td>
<td>3.8%</td>
<td>908</td>
<td>2.6%</td>
<td>1,273</td>
<td>0.7%</td>
</tr>
<tr>
<td>0</td>
<td>0.0%</td>
<td>457</td>
<td>2.7%</td>
<td>925</td>
<td>2.5%</td>
<td>1,277</td>
<td>0.5%</td>
</tr>
<tr>
<td>17</td>
<td>0.5%</td>
<td>474</td>
<td>2.1%</td>
<td>930</td>
<td>3.9%</td>
<td>1,293</td>
<td>1.5%</td>
</tr>
<tr>
<td>44</td>
<td>0.7%</td>
<td>487</td>
<td>2.8%</td>
<td>931</td>
<td>4.3%</td>
<td>1,297</td>
<td>0.2%</td>
</tr>
<tr>
<td>60</td>
<td>1.0%</td>
<td>494</td>
<td>2.9%</td>
<td>944</td>
<td>4.1%</td>
<td>1,308</td>
<td>2.3%</td>
</tr>
<tr>
<td>78</td>
<td>1.3%</td>
<td>500</td>
<td>3.1%</td>
<td>950</td>
<td>4.4%</td>
<td>1,332</td>
<td>0.8%</td>
</tr>
<tr>
<td>85</td>
<td>1.2%</td>
<td>501</td>
<td>4.0%</td>
<td>960</td>
<td>4.4%</td>
<td>1,335</td>
<td>1.3%</td>
</tr>
<tr>
<td>91</td>
<td>0.8%</td>
<td>507</td>
<td>1.3%</td>
<td>982</td>
<td>2.5%</td>
<td>1,349</td>
<td>2.5%</td>
</tr>
<tr>
<td>93</td>
<td>1.5%</td>
<td>511</td>
<td>2.0%</td>
<td>985</td>
<td>3.1%</td>
<td>1,353</td>
<td>2.4%</td>
</tr>
<tr>
<td>100</td>
<td>1.0%</td>
<td>551</td>
<td>7.6%</td>
<td>997</td>
<td>2.5%</td>
<td>1,371</td>
<td>1.8%</td>
</tr>
<tr>
<td>124</td>
<td>1.5%</td>
<td>577</td>
<td>4.2%</td>
<td>1,003</td>
<td>0.0%</td>
<td>1,391</td>
<td>0.8%</td>
</tr>
<tr>
<td>153</td>
<td>0.9%</td>
<td>602</td>
<td>4.1%</td>
<td>1,007</td>
<td>0.3%</td>
<td>1,393</td>
<td>2.4%</td>
</tr>
<tr>
<td>154</td>
<td>0.9%</td>
<td>608</td>
<td>4.7%</td>
<td>1,027</td>
<td>1.2%</td>
<td>1,412</td>
<td>1.5%</td>
</tr>
<tr>
<td>176</td>
<td>1.5%</td>
<td>611</td>
<td>1.1%</td>
<td>1,032</td>
<td>1.6%</td>
<td>1,441</td>
<td>1.4%</td>
</tr>
<tr>
<td>183</td>
<td>1.0%</td>
<td>622</td>
<td>1.9%</td>
<td>1,040</td>
<td>1.5%</td>
<td>1,449</td>
<td>1.6%</td>
</tr>
<tr>
<td>191</td>
<td>1.0%</td>
<td>632</td>
<td>3.2%</td>
<td>1,047</td>
<td>1.4%</td>
<td>1,463</td>
<td>1.9%</td>
</tr>
<tr>
<td>207</td>
<td>1.5%</td>
<td>642</td>
<td>2.9%</td>
<td>1,052</td>
<td>1.9%</td>
<td>1,470</td>
<td>2.5%</td>
</tr>
<tr>
<td>219</td>
<td>1.5%</td>
<td>643</td>
<td>1.4%</td>
<td>1,067</td>
<td>0.8%</td>
<td>1,476</td>
<td>2.9%</td>
</tr>
<tr>
<td>235</td>
<td>1.0%</td>
<td>653</td>
<td>1.3%</td>
<td>1,071</td>
<td>0.9%</td>
<td>1,495</td>
<td>2.7%</td>
</tr>
<tr>
<td>252</td>
<td>1.4%</td>
<td>663</td>
<td>2.9%</td>
<td>1,077</td>
<td>3.5%</td>
<td>1,524</td>
<td>1.6%</td>
</tr>
<tr>
<td>261</td>
<td>1.6%</td>
<td>681</td>
<td>3.9%</td>
<td>1,085</td>
<td>3.4%</td>
<td>1,553</td>
<td>1.8%</td>
</tr>
<tr>
<td>275</td>
<td>2.0%</td>
<td>682</td>
<td>2.4%</td>
<td>1,112</td>
<td>2.0%</td>
<td>1,561</td>
<td>3.0%</td>
</tr>
<tr>
<td>293</td>
<td>0.7%</td>
<td>695</td>
<td>1.7%</td>
<td>1,137</td>
<td>0.3%</td>
<td>1,566</td>
<td>3.2%</td>
</tr>
<tr>
<td>320</td>
<td>1.0%</td>
<td>724</td>
<td>5.5%</td>
<td>1,141</td>
<td>0.2%</td>
<td>1,579</td>
<td>4.2%</td>
</tr>
<tr>
<td>330</td>
<td>1.4%</td>
<td>745</td>
<td>6.3%</td>
<td>1,142</td>
<td>0.8%</td>
<td>1,587</td>
<td>4.8%</td>
</tr>
<tr>
<td>337</td>
<td>1.2%</td>
<td>747</td>
<td>2.5%</td>
<td>1,143</td>
<td>3.0%</td>
<td>1,612</td>
<td>5.1%</td>
</tr>
<tr>
<td>343</td>
<td>1.6%</td>
<td>776</td>
<td>0.0%</td>
<td>1,160</td>
<td>2.6%</td>
<td>1,622</td>
<td>4.6%</td>
</tr>
<tr>
<td>356</td>
<td>2.6%</td>
<td>787</td>
<td>0.4%</td>
<td>1,169</td>
<td>3.0%</td>
<td>1,631</td>
<td>2.4%</td>
</tr>
<tr>
<td>360</td>
<td>1.7%</td>
<td>798</td>
<td>0.3%</td>
<td>1,187</td>
<td>2.3%</td>
<td>1,650</td>
<td>3.3%</td>
</tr>
<tr>
<td>365</td>
<td>1.9%</td>
<td>803</td>
<td>2.5%</td>
<td>1,189</td>
<td>4.1%</td>
<td>1,660</td>
<td>3.2%</td>
</tr>
<tr>
<td>378</td>
<td>2.0%</td>
<td>830</td>
<td>0.5%</td>
<td>1,221</td>
<td>2.6%</td>
<td>1,697</td>
<td>1.9%</td>
</tr>
<tr>
<td>384</td>
<td>1.8%</td>
<td>838</td>
<td>1.3%</td>
<td>1,223</td>
<td>3.0%</td>
<td>1,701</td>
<td>2.3%</td>
</tr>
<tr>
<td>396</td>
<td>2.3%</td>
<td>841</td>
<td>1.1%</td>
<td>1,242</td>
<td>1.6%</td>
<td>1,719</td>
<td>2.4%</td>
</tr>
</tbody>
</table>
Figure 32. FEA Mode Shapes for the Shell Without the Broad Side: 60 Hz to 663 Hz
Figure 33. FEA Mode Shapes for the Shell Without the Broad Side: 776 Hz to 1,495 Hz
Following the simulation of the shell without the broad side, the removed side was added to the shell and another simulation was run. Again, surface meshing and the multi pass adaptive method were used to generate a solution. The maximum polynomial order was set to nine and the convergence was set to 10%. Like the previous tests, the simulation frequency ranges were staggered to improve convergence. Table 8 shows most of the calculated natural frequencies and the percent convergence. As shown, all of the calculated natural frequencies have a rate of convergence below 10%. The natural frequencies also fall within the bounds set using Blevins’s methods. Figure 34 and Figure 35 show some of the mode shapes calculated at or near the harmonics of 60 Hz. The mode shapes for the broad sides look as expected until approximately 600 Hz. After 600 Hz, the half-wave pattern breaks down and the mode shapes do not have a pattern. The narrow side also possesses the checkerboard pattern until approximately 960 Hz. The results from the FEA analysis show that the complete shell has natural frequencies near most of the frequencies of acoustic emission. The FEA analysis also shows that several of the mode shapes have large displacements on the narrow side and smaller displacements on the broad side. This is worth noting because, as discussed in Chapter 5, this is where the largest excitation occurs.

The results of the FEA simulations, like the models made using Blevins’s methods, are somewhat hard to use for this application. The simulation provides good visual understanding to the shapes that develop, but the gain in magnitude at these resonant frequencies is not clearly given, through a Bode plot, for example. Therefore, it is difficult to determine from the FEA analysis alone which resonant frequencies are dominant.
Table 8
FEA Calculated Natural Frequencies for the Complete Shell

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Convergence (%)</th>
<th>Frequency (Hz)</th>
<th>Convergence (%)</th>
<th>Frequency (Hz)</th>
<th>Convergence (%)</th>
<th>Frequency (Hz)</th>
<th>Convergence (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0%</td>
<td>451</td>
<td>4.2%</td>
<td>821</td>
<td>2.4%</td>
<td>1,166</td>
<td>3.1%</td>
</tr>
<tr>
<td>0</td>
<td>0.0%</td>
<td>464</td>
<td>2.3%</td>
<td>833</td>
<td>2.2%</td>
<td>1,184</td>
<td>2.5%</td>
</tr>
<tr>
<td>0</td>
<td>0.0%</td>
<td>468</td>
<td>2.1%</td>
<td>836</td>
<td>3.4%</td>
<td>1,189</td>
<td>2.4%</td>
</tr>
<tr>
<td>0</td>
<td>0.0%</td>
<td>470</td>
<td>3.0%</td>
<td>859</td>
<td>2.0%</td>
<td>1,191</td>
<td>2.8%</td>
</tr>
<tr>
<td>0</td>
<td>0.0%</td>
<td>484</td>
<td>1.9%</td>
<td>860</td>
<td>2.2%</td>
<td>1,200</td>
<td>2.8%</td>
</tr>
<tr>
<td>0</td>
<td>0.0%</td>
<td>485</td>
<td>2.0%</td>
<td>866</td>
<td>3.1%</td>
<td>1,209</td>
<td>2.7%</td>
</tr>
<tr>
<td>61</td>
<td>0.4%</td>
<td>487</td>
<td>4.8%</td>
<td>867</td>
<td>4.1%</td>
<td>1,217</td>
<td>2.6%</td>
</tr>
<tr>
<td>87</td>
<td>0.6%</td>
<td>508</td>
<td>0.4%</td>
<td>880</td>
<td>3.9%</td>
<td>1,236</td>
<td>2.2%</td>
</tr>
<tr>
<td>96</td>
<td>0.3%</td>
<td>528</td>
<td>2.1%</td>
<td>888</td>
<td>3.8%</td>
<td>1,239</td>
<td>3.3%</td>
</tr>
<tr>
<td>115</td>
<td>0.4%</td>
<td>533</td>
<td>1.6%</td>
<td>910</td>
<td>2.3%</td>
<td>1,245</td>
<td>3.4%</td>
</tr>
<tr>
<td>152</td>
<td>0.2%</td>
<td>539</td>
<td>3.2%</td>
<td>943</td>
<td>0.3%</td>
<td>1,263</td>
<td>0.5%</td>
</tr>
<tr>
<td>154</td>
<td>0.2%</td>
<td>564</td>
<td>2.0%</td>
<td>949</td>
<td>2.3%</td>
<td>1,280</td>
<td>0.0%</td>
</tr>
<tr>
<td>173</td>
<td>0.4%</td>
<td>567</td>
<td>4.0%</td>
<td>956</td>
<td>2.5%</td>
<td>1,288</td>
<td>2.1%</td>
</tr>
<tr>
<td>192</td>
<td>0.3%</td>
<td>588</td>
<td>0.8%</td>
<td>960</td>
<td>3.5%</td>
<td>1,290</td>
<td>2.6%</td>
</tr>
<tr>
<td>194</td>
<td>0.7%</td>
<td>592</td>
<td>2.8%</td>
<td>972</td>
<td>2.7%</td>
<td>1,317</td>
<td>1.0%</td>
</tr>
<tr>
<td>217</td>
<td>0.8%</td>
<td>598</td>
<td>2.7%</td>
<td>976</td>
<td>3.5%</td>
<td>1,337</td>
<td>0.2%</td>
</tr>
<tr>
<td>223</td>
<td>0.2%</td>
<td>609</td>
<td>1.4%</td>
<td>980</td>
<td>4.3%</td>
<td>1,343</td>
<td>0.5%</td>
</tr>
<tr>
<td>241</td>
<td>0.3%</td>
<td>611</td>
<td>3.9%</td>
<td>984</td>
<td>4.9%</td>
<td>1,374</td>
<td>0.2%</td>
</tr>
<tr>
<td>254</td>
<td>0.5%</td>
<td>622</td>
<td>4.8%</td>
<td>1,003</td>
<td>0.3%</td>
<td>1,381</td>
<td>0.4%</td>
</tr>
<tr>
<td>256</td>
<td>0.5%</td>
<td>647</td>
<td>0.9%</td>
<td>1,011</td>
<td>0.1%</td>
<td>1,397</td>
<td>0.2%</td>
</tr>
<tr>
<td>274</td>
<td>0.4%</td>
<td>652</td>
<td>2.9%</td>
<td>1,021</td>
<td>0.0%</td>
<td>1,407</td>
<td>0.2%</td>
</tr>
<tr>
<td>278</td>
<td>0.5%</td>
<td>657</td>
<td>4.0%</td>
<td>1,036</td>
<td>0.1%</td>
<td>1,412</td>
<td>1.0%</td>
</tr>
<tr>
<td>318</td>
<td>1.3%</td>
<td>662</td>
<td>4.4%</td>
<td>1,038</td>
<td>0.2%</td>
<td>1,431</td>
<td>0.3%</td>
</tr>
<tr>
<td>326</td>
<td>0.6%</td>
<td>683</td>
<td>3.3%</td>
<td>1,051</td>
<td>0.4%</td>
<td>1,440</td>
<td>1.9%</td>
</tr>
<tr>
<td>339</td>
<td>0.9%</td>
<td>684</td>
<td>5.0%</td>
<td>1,054</td>
<td>0.7%</td>
<td>1,453</td>
<td>2.0%</td>
</tr>
<tr>
<td>340</td>
<td>1.4%</td>
<td>694</td>
<td>6.6%</td>
<td>1,059</td>
<td>2.7%</td>
<td>1,460</td>
<td>2.0%</td>
</tr>
<tr>
<td>350</td>
<td>1.2%</td>
<td>708</td>
<td>8.1%</td>
<td>1,064</td>
<td>2.4%</td>
<td>1,466</td>
<td>3.1%</td>
</tr>
<tr>
<td>357</td>
<td>1.6%</td>
<td>738</td>
<td>4.6%</td>
<td>1,065</td>
<td>2.5%</td>
<td>1,481</td>
<td>2.4%</td>
</tr>
<tr>
<td>362</td>
<td>1.1%</td>
<td>743</td>
<td>5.4%</td>
<td>1,081</td>
<td>1.2%</td>
<td>1,488</td>
<td>3.8%</td>
</tr>
<tr>
<td>374</td>
<td>2.0%</td>
<td>756</td>
<td>1.2%</td>
<td>1,096</td>
<td>1.0%</td>
<td>1,494</td>
<td>4.0%</td>
</tr>
<tr>
<td>388</td>
<td>1.3%</td>
<td>763</td>
<td>1.0%</td>
<td>1,101</td>
<td>1.4%</td>
<td>1,509</td>
<td>3.9%</td>
</tr>
<tr>
<td>393</td>
<td>2.3%</td>
<td>771</td>
<td>1.4%</td>
<td>1,123</td>
<td>0.5%</td>
<td>1,513</td>
<td>4.1%</td>
</tr>
<tr>
<td>400</td>
<td>1.1%</td>
<td>775</td>
<td>1.9%</td>
<td>1,132</td>
<td>1.2%</td>
<td>1,549</td>
<td>3.5%</td>
</tr>
<tr>
<td>403</td>
<td>2.2%</td>
<td>788</td>
<td>2.7%</td>
<td>1,139</td>
<td>1.0%</td>
<td>1,557</td>
<td>3.9%</td>
</tr>
<tr>
<td>411</td>
<td>4.1%</td>
<td>794</td>
<td>2.2%</td>
<td>1,147</td>
<td>1.8%</td>
<td>1,571</td>
<td>3.9%</td>
</tr>
<tr>
<td>422</td>
<td>1.9%</td>
<td>799</td>
<td>2.7%</td>
<td>1,149</td>
<td>2.6%</td>
<td>1,580</td>
<td>3.7%</td>
</tr>
<tr>
<td>439</td>
<td>4.4%</td>
<td>813</td>
<td>2.8%</td>
<td>1,157</td>
<td>3.4%</td>
<td>1,592</td>
<td>3.6%</td>
</tr>
</tbody>
</table>
Figure 34. FEA Mode Shapes for the Shell: 61 Hz to 662 Hz
Figure 35. FEA Mode Shapes for the Shell: 836 Hz to 1,440 Hz
The accuracy of both models is also questionable. Achieving correct results is greatly dependent on the model dimensions and material parameters, especially for plate thicknesses as small as those used for the shell. This is apparent from Equation 17 and Equation 19, in which the natural frequencies of the plate are a function of the plate thickness cubed and the ratio of the length and width squared. Small inaccuracies in measured dimensions can result in large inaccuracies in calculated natural frequencies.

As discussed previously, analysis of the capacitor as a whole was not performed. Modeling the capacitor would require a much more complicated model to be developed, which would have to take into account fluid coupling through CFD modeling. The internal elements would also have to be modeled, which would require a great deal of effort to accurately simulate because they are not fully constrained inside the shell. Software is available that could be used to model both of these factors, but this is left as an area for future work.

Conclusions

Modal analysis of the capacitor shell was performed using theoretical methods. The shell was first modeled using plate theory described by Blevins. The sides of the shell were considered as both clamped and simply supported plates for the analysis. Considering both conditions set the high and low limits for the natural frequencies. Using Blevins’s methods, the natural frequencies of the shell’s broad sides and narrow sides were calculated. The mode shapes of these resonant frequencies were determined to have a checkerboard pattern.

Because Blevins’s methods are difficult to apply to plates with mixed boundary conditions, FEA analysis was used to model the shell. First, a plate with free boundary conditions was modeled in order to learn the software. The natural frequencies of the plate are presented as well as the associated mode shapes. Next, the shell without one of the broad sides
was modeled. The calculated natural frequencies to 1,600 Hz are given along with the mode shapes of the frequencies near odd harmonics of 60 Hz. Most of the mode shapes of the broad sides have elements of the checkerboard pattern determined using Blevins’s method. Finally, the entire shell was modeled. The natural frequencies calculated with the FEA software fell within the bounds set with Blevins’s methods. The mode shapes calculated have the checkerboard pattern until 600 Hz, at which point the mode shapes are almost random. This could be caused by the number of elements used for the model. Increasing the number of elements used may show patterns in the higher frequency mode shapes, but this is left to future research.

Analysis of the capacitor as a whole was not performed. Modeling the capacitor would require a much more complicated model to be developed that could model the effects of the large internal mass of the capacitive elements and the coupling from the dielectric fluid. If more time was available, an accurate model of the capacitor could be developed in future research.
CHAPTER 5
VIBRATION ANALYSIS

As discussed in Chapter 2, the dominant frequencies of acoustic generation at SUB 2 are at the 2\textsuperscript{nd}, 4\textsuperscript{th}, 6\textsuperscript{th}, and 12\textsuperscript{th} harmonics of 60 Hz. At SUB 1, the dominant frequencies are at the 2\textsuperscript{nd}, 14\textsuperscript{th}, 22\textsuperscript{nd}, and 24\textsuperscript{th} harmonics, with the 22\textsuperscript{nd} and 24\textsuperscript{th} having the largest magnitude [11]. However, the harmonic current near the 22\textsuperscript{nd} and 24\textsuperscript{th} harmonic is less than the current near other frequencies. Cox and Guan proposed that the electric forces generated within the capacitor can excite structural natural frequencies, causing this behavior [5]. From this observation, it was decided that experimental vibration analysis was needed to determine if structural natural frequencies are being excited. Initially, impact hammer testing similar to that performed by Cox and Guan was planned to determine these natural frequencies. However a scanning laser vibrometer at The University of Alabama Structural Acoustics Laboratory was made available and used for testing. This type of testing has advantages over impact hammer testing. The scanning laser vibrometer cannot only determine structural natural frequencies but will also display the mode shapes at these natural frequencies. With additional resources provided, including an electromagnetic shaker, vibration testing methods superior to those available to Cox and Guan were utilized.

During the initial stages of testing, a free plate was analyzed using the vibrometer system in order to learn about the software and proper test setup. The results are shown and match well
with the FEA model discussed in Chapter 4. Next, the working capacitor was tested. The capacitor was excited over a frequency range using an electromagnetic shaker and the response was recorded at several points using the laser vibrometer. The capacitor was excited on both the narrow side and broad side. Several adjustments were made during testing, including the number of measuring points, the excitation point, and the excitation sweep rate. Tests were also run to measure the response of one wall to the excitation on another wall. The results of all these tests are discussed in detail.

The failed capacitor was also tested to determine how the liquid dielectric affects the frequency response. The measured response of the drained capacitor is compared to the response of a similar working capacitor and the results show that the dielectric has a large damping effect. Finally, the shell of the failed capacitor was tested. The experimental results are compared to the FEA model discussed in Chapter 4 and the results are similar. To conclude the chapter, the validity of the test methods is discussed. The methods used are also compared to those used by Cox and Guan during their experiment.

Setup

The scanning laser vibrometer measures the two-dimensional distribution of vibration velocities on the basis of laser interferometry [15]. The scanning head emits a laser beam that first strikes a mirror that is thinly coated with silver, allowing half of the beam to pass through while the other half is reflected. The reflected half is used as a reference. The half that passes through strikes the object being tested and is reflected back [17]. The measurement beam has a Doppler frequency shift that is a function of the test piece’s surface velocity. By comparing the two signals, the relative velocities of the test piece as its being excited can be determined [2].
The scanning laser vibrometer used was the Polytec PSV 300. The package includes a Polytec OFV-056 Vibrometer Scanning Head, a Polytec OFV-3001S Vibrometer Controller, a Polytec PSV-Z-040H Junction Box, and the workstation (a PC). A typical setup is shown in Figure 36 and the components of the system are shown in Figure 37. The scanning head consists of the interferometer, the scanners to deflect the laser beam and a video camera to visualize the measurement object. The interferometer signal is decoded in the controller with a velocity decoder which generates an analog voltage signal proportional to the vibration velocity. This measurement data is digitally recorded in the workstation. The software controls the data acquisition and allows the user to easily evaluate data. The junction box connects the system components and provides the interfaces for peripheral devices [16].

Figure 36. Typical Scanning Laser Vibrometer Setup
Figure 37. Scanning Laser Vibrometer System Components [16]

The first step of setting up the vibration analysis was preparing the test piece for excitation. This setup is shown in Figure 38 for a typical capacitor test. The capacitor was placed across two sawhorses. For all of the vibration analysis, an electromagnetic shaker was used to excite the test piece. The shaker was hung from a support structure. In Figure 38, the support structure used was a piece of steel channel bolted to the capacitor support. The shaker was set up to be driven by a signal from the Polytec software which was enlarged by an amplifier. For the excitation reference, a PCB208-CO2-SN-19011 impedance head was used. The impedance head measures the force generated by the shaker with an output of 50 mV/lbf. The impedance head was connected to the end of the shaker stinger. A mounting foot was then connected to the other end of the impedance head and glued to the testing surface.

After preparing the test subject for excitation, the measuring components were set up. The output of the impedance head was connected to an amplifier. For all of the tests, the gain of this amplifier was set to 10. The output of the amplifier was connected to the reference channel of the junction box. In order to get the best measurements, the distance between the laser head and the test specimen needs to be at a multiple of 205 millimeters [15]. The laser was positioned
approximately 8 feet from the test subject. The complete setup is shown in Figure 39 and Figure 40.

Figure 38. Experimental Excitation Setup

Figure 39. Experimental Measurement Setup
To increase the accuracy of the test measurements, several test characteristics can be adjusted. First, care must be taken to ensure the laser is properly focused. This has a large effect on improving the signal to noise ratio. Depending on the surface finish and roughness of the test piece, the laser may be hard to focus. The signal can be improved by painting the test piece with a reflective coating. Improving the reflectivity of the measurement surface increases the strength of the return signal. Speckle tracking filters are also used to improve the signal to noise ratio. These filters reduce noise by bypassing the intermittent periods when no laser light is reflected back. The next factor that can be adjusted is the number of scan points. By increasing the density of the scan point grid, the measurements will better represent the system. The excitation sweep rate can also be adjusted. This is the amount of time taken to sweep the excitation over the desired frequency range. If the sweep rate is too fast, the test specimen may not have enough time to fully respond at each frequency. The most important adjustment that can be made is the type and number of averages performed on the data. Averaging reduces the amount of noise.
present in the data. For all of the tests performed, complex averaging was used, which uses both
the magnitude and phase of the measurement for averaging [14]. The drawback for adjusting all
of these test characteristics is the amount of time required per test. For example, doubling the
amount of scan points doubles the amount of time required for testing. The same is true for the
averaging and the sweep time. Therefore, a balance between measurement accuracy and minimal
test time must be found. This is discussed throughout the chapter.

Using the velocity measurements recorded, the Polytec software can display the
displacement and acceleration of the test points. Based on some research into the acoustics of
plates, it was determined that the noise generated by plates is most closely related to the velocity
of the plate [7]. Therefore, the velocity measurements recorded are the most applicable to this
study. The software displays the magnitude and phase of the measured frequency response.
However, to export this information, a file must be manually created for each point. Because
most of the tests ran had over 200 scan points, this would require a significant effort. This would
also require a great deal of data processing. An alternate method that can be used is displaying
the “average response” of the system. This is the average of the magnitude response measured at
each point and gives a good estimate of the overall response of the system. The Polytec software
also has the option of displaying the frequency response function (FRF) of the system, which is
the frequency response of the system per input [14]. For this study, the mobility (output velocity
per input force) of the system is of interest. All of the experimental data that follows for the
capacitor is given in terms of the average mobility.
Testing

A simple free plate was first tested to gain experience with the equipment and software. Also, because there are several resources available on the natural frequencies and mode shapes of plates, proper test setup could be learned by comparing experimental results to analytical results. The plate was suspended from a saw horse using a bungee cord. An electromagnetic shaker was used for the excitation. The shaker was suspended from another sawhorse, also using bungee cords, and the two were coupled using a stinger. The laser vibrometer was positioned to measure the vibration of the side of the plate opposite the stinger. The plate was excited using white noise with a component range of 50 Hz to 5 kHz, and the response of the plate was measured. An accelerometer was used for the input reference. For the test, 99 points were scanned and the number of averages used was three. Figure 42 shows the frequency response of the plate. Figure 43 shows the corresponding mode shapes measured by the vibrometer. Table 9 shows the comparison between the natural frequencies calculated in Chapter 4 for the free plate and the experimental results. The experimental frequencies match those determined using the FEA software to within 10%. However, some of the natural frequencies calculated using the FEA software did not show up in the experimental results. After inspecting the measured data, it was determined that several of the data points were not valid. These points had a significant amount of noise present which distorted the results. However, the test setup had already been dismantled and it was decided not to retest.
Figure 41. Free Plate Test Setup

Figure 42. Experimental Frequency Response for the Free Plate
Figure 43. Experimental Mode Shapes for the Free Plate
After getting reasonable results from testing the plate, it was decided to start testing the capacitors. There were three goals set before testing began. The primary goal was to determine the natural frequencies and mode shapes of the complete capacitor. Because both a fluid filled capacitor and an empty capacitor were available for testing, the second goal was to determine the effect of fluid loading on the structural resonances. It was also known that the empty capacitor was going to be cut open, so the third goal was to test the outer shell of the capacitor. It should also be noted that, at this point in the project, little was known about the internal construction of the capacitor. The only clues to the construction were found in Figure 8, which shows very little detail of the internal construction. The figure does show that the capacitive elements are in a vertical arrangement. This led to the assumption that the shell of the capacitor is mainly excited on the narrow walls. However, it was decided to excite both the broad and narrow walls and measure the response.

Table 9.
Free Plate Natural Frequency Comparison

<table>
<thead>
<tr>
<th>Experimental (Hz)</th>
<th>FEA (Hz)</th>
<th>Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>411</td>
<td>396</td>
<td>4%</td>
</tr>
<tr>
<td>537</td>
<td>530</td>
<td>1%</td>
</tr>
<tr>
<td>791</td>
<td>811</td>
<td>3%</td>
</tr>
<tr>
<td>998</td>
<td>965</td>
<td>3%</td>
</tr>
<tr>
<td>1,135</td>
<td>1,097</td>
<td>3%</td>
</tr>
<tr>
<td>1,473</td>
<td>1,540</td>
<td>5%</td>
</tr>
<tr>
<td>1,773</td>
<td>1,876</td>
<td>6%</td>
</tr>
<tr>
<td>2,116</td>
<td>1,894</td>
<td>10%</td>
</tr>
<tr>
<td>2,255</td>
<td>2,170</td>
<td>3%</td>
</tr>
<tr>
<td></td>
<td>2,487</td>
<td>10%</td>
</tr>
<tr>
<td>-</td>
<td>2,942</td>
<td>-</td>
</tr>
<tr>
<td>-</td>
<td>2,969</td>
<td>-</td>
</tr>
<tr>
<td>3,068</td>
<td>3,179</td>
<td>4%</td>
</tr>
<tr>
<td>3,476</td>
<td>3,425</td>
<td>1%</td>
</tr>
</tbody>
</table>
The first tests were conducted by exciting and measuring the broad side. Initially, the capacitor was excited near the center, as shown in Figure 38. The sweep rate was set to 10 seconds for faster testing. The frequency was swept from 100 Hz to 4 kHz. The number of scan points used for the first tests was relatively high (approximately 390 points). This was done to ensure all motion was seen. Figure 44 shows the test setup and scan point density. Figure 45 shows the corresponding frequency response spectrum. The figure clearly shows a large increase in the response gain between 100 Hz and 500 Hz. The magnitude of the response then begins to decrease at approximately 1,500 Hz. Peaks in the response are seen at 472 Hz, 725 Hz, 1,136 Hz, 1,350 Hz, and 1,480 Hz. These peaks are close to the 8th, 12th, 19th, 22nd, and 24th harmonics of 60 Hz. However, the largest peaks are not near the 22nd and 24th harmonic, as expected, but are near the 8th and 12th harmonic.

As discussed previously, one major advantage of testing with a laser vibrometer is the ability to determine not only the resonant frequencies but also the mode shapes of the test subject. After examining the results of the first test, no standing mode shapes were seen. The

![Initial Scan Point Density](image-url)
waves propagate away from the excitation point and the magnitude of the response decreases as it propagates. It was decided that the capacitor should be excited at several locations because the internal support structure was unknown as well as the location of any node lines. This would also give a better representation of the natural frequencies of the system.

To change the location of excitation, the setup shown in Figure 38 was slightly modified. Instead of hanging the shaker from the steel channel, another support structure was used. This allowed the location of the shaker to be easily adjusted. The locations are shown in Figure 46 and given in Table 10. The frequency was swept from 100 Hz to 4 kHz for all of these tests with an associated sweep rate of 10 seconds. The scan point density used for the previous test was not changed. Figure 47 shows the average frequency response at these locations from 100 Hz to 2 kHz, which is region of interest for audible noise generation. The spectrums are considerably different depending on the location of excitation. However, there are three frequency ranges where common peaks are seen. These ranges are 400 Hz to 600 Hz, 700 Hz to 800 Hz, and
1,100 Hz to 1,200 Hz. As seen in Figure 47, the largest response occurs around 1,150 Hz. This is interesting because, according to Dr. Haskew’s EMTP tests, there is also an electrical resonance frequency in this range. No standing mode shapes were seen in any of these tests. Again, the waves propagate away from the excitation source and no standing waves are seen.

![Figure 46. Initial Excitation Locations for the Broad Side](image)

Table 10: Initial Test Locations for the Broad Side

<table>
<thead>
<tr>
<th>Test</th>
<th>X (Inches)</th>
<th>Y (Inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3 1/4</td>
<td>10 1/4</td>
</tr>
<tr>
<td>2</td>
<td>10 3/4</td>
<td>7 1/2</td>
</tr>
<tr>
<td>3</td>
<td>2 1/2</td>
<td>17 3/4</td>
</tr>
<tr>
<td>4</td>
<td>3 3/4</td>
<td>20</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td>17 3/4</td>
</tr>
<tr>
<td>6</td>
<td>9 1/4</td>
<td>20 1/2</td>
</tr>
</tbody>
</table>
Figure 47. Average Frequency Response for the Broad Side at Different Excitation Locations

When the capacitor is in operation, the excitation is distributed. After seeing how the location of excitation has a large effect on the response from the previous tests, it was decided that tests should be run while moving the excitation location in a structured pattern instead of the random pattern shown in Figure 46. This would give the best representation of the system response while it is operating. It was assumed for this test that the capacitor is nearly symmetric in the Y direction of Figure 46, so only half the capacitor would need to be tested. A grid of 15 points was marked on the lower half of the capacitor. The test points are shown in Figure 48 and given in Table 11. For all of the tests, the frequency was swept from 10 Hz to 2.5 kHz. The sweep rate used was 20 seconds and the data was averaged 40 times. The number of scan points used was approximately 220 for every test. Figure 49, Figure 50, and Figure 51 show the frequency response function at each point. Averaging the response at all these points yields the frequency response function in Figure 52. The average response shows a large increase in gain from 100 to 300 Hz, which holds until approximately 500 Hz. After this, there is a region of gain which is almost linear until 2 kHz. The responses at 1,320 Hz and 1,440 Hz fall near a valley, but
the magnitude at these frequencies is still large than frequencies at 500 Hz and below. All of the figures show some sudden spikes that should not be considered as they are the result of electrical noise.

Figure 48. Structured Excitation Locations for the Broad Side

Table 11
Structured Excitation Locations for the Broad Side

<table>
<thead>
<tr>
<th>Test</th>
<th>X</th>
<th>Y</th>
<th>Test</th>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-)</td>
<td>(Inches)</td>
<td>(Inches)</td>
<td>(-)</td>
<td>(Inches)</td>
<td>(Inches)</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>6 1/2</td>
<td>9</td>
<td>18 1/2</td>
<td>4 1/2</td>
</tr>
<tr>
<td>2</td>
<td>7 1/2</td>
<td>6 1/2</td>
<td>10</td>
<td>23 5/8</td>
<td>4 1/2</td>
</tr>
<tr>
<td>3</td>
<td>13</td>
<td>6 1/2</td>
<td>11</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>18 1/2</td>
<td>6 1/2</td>
<td>12</td>
<td>7 1/2</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>23 5/8</td>
<td>6 1/2</td>
<td>13</td>
<td>13</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>4 1/2</td>
<td>14</td>
<td>18 1/2</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>7 1/2</td>
<td>4 1/2</td>
<td>15</td>
<td>23 5/8</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>13</td>
<td>4 1/2</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 49. Average Frequency Response at Test Locations 1-5

Figure 50. Average Frequency Response at Test Locations 6-10
Figure 51. Average Frequency Response at Test Locations 11-15

Figure 52. Average of the Responses for Test Points 1-15
For each of the previous tests, the capacitor was excited on the broad side and the response was recorded. For the next set of testing, the capacitor was excited on the narrow side. The capacitor was supported so that the narrow side could be directly measured with the laser. The shaker was suspended from the support structure using bungee cords. Figure 53 shows an example of the test setup. The location of excitation was moved five times during this set of tests. The excitation locations are shown in Figure 54 and are given in Table 12. The frequency was swept from 10 Hz to 2.5 kHz for each test using a sweep rate of 20 seconds. The frequency spectrums of the tests are shown in Figure 55. As shown, most of the measured responses have some peaks in common. These peaks are at approximately 750 Hz, 950 Hz, and 1.6 kHz. However, there are no peaks in the frequencies of interest, most notably at 22nd and 24th harmonic. This is more clearly shown in Figure 56, which shows the average of the measured responses.

Figure 53. Narrow Side Test Setup
Figure 54. Narrow Side Excitation Locations

Table 12
Narrow Side Excitation Locations

<table>
<thead>
<tr>
<th>Test</th>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-)</td>
<td>(Inches)</td>
<td>(Inches)</td>
</tr>
<tr>
<td>1</td>
<td>2 5/8</td>
<td>3 1/2</td>
</tr>
<tr>
<td>2</td>
<td>9 1/8</td>
<td>3 3/4</td>
</tr>
<tr>
<td>3</td>
<td>13 1/4</td>
<td>3 3/4</td>
</tr>
<tr>
<td>4</td>
<td>17 1/4</td>
<td>3 1/2</td>
</tr>
<tr>
<td>5</td>
<td>21 1/2</td>
<td>3 1/2</td>
</tr>
</tbody>
</table>

Figure 55. Average Frequency Response for Each Narrow Side Location
Because it was believed that, under operating conditions, the capacitor is mainly excited on the narrow side, the response of the broad side from excitation on the narrow side was desired. The capacitor was supported so the broad side could be measured by the laser. The shaker was hung from two support structures using bungee cords. The shaker was rotated vertically and the mounting foot was glued to the narrow side. Some of the setup can be seen in Figure 57. The frequency response function for this test is shown in Figure 58. The results show that, under these test conditions, the largest response is seen at approximately 1,660 Hz. Peaks in the response are also seen at 720 Hz, 1,150 Hz, and 2,200 Hz. However, when comparing Figure 52 to Figure 58, it is clear that the magnitude of the response measured on the broad side to excitation from the narrow side is minimal. The largest magnitude seen in Figure 58 is less than 10 percent of the largest magnitude shown in Figure 52.
After performing the previous test, it was decided that the capacitor should also be excited on the broad side while measuring the response on the narrow side. However, after the previous test, it was learned that several sets of mirrors were available. These mirrors can be
used to reflect the laser onto a surface that cannot be measured directly by the vibrometer. This greatly simplified the test setup and allows multiple surfaces to be scanned during the same test. Figure 59 shows the test setup using the mirrors. The mirror was positioned above the narrow side of the capacitor and was angled at 45 degrees. Figure 60 shows the average frequency response of the narrow side. When compared to the response shown in Figure 55, it can be seen that excitation on the broad side has a considerable effect on the response of the narrow side. The largest response shown in Figure 60 is approximately 60% of the largest magnitude measured in Figure 55. However, there are no peaks at the 22\textsuperscript{nd} and 24\textsuperscript{th} harmonic of 60 Hz.

Figure 59. Test Setup for Measuring the Narrow Side Response from Broad Side Excitation

As discussed previously, two capacitors were provided by the utility provider for testing. One of the capacitors had already been drained of the dielectric fluid and was mainly provided
Figure 60. Average Frequency Response for the Narrow Side with Broad Side Excitation for dissection. Because both an empty and fluid filled capacitor were available, it was decided that the empty capacitor would also be tested before it was cut open. The goal of this was to determine the effect of fluid loading on the response of the capacitor. After examining both capacitors, it was noticed that the two capacitors had the same ratings, but their dimensions were different. The internal construction of the two was assumed to be similar.

The empty capacitor was first tested on the broad side. The excitation was swept from 100 Hz to 4 kHz and the response was measured. Figure 61 shows the response of the capacitor with no dielectric and the response of the capacitor with dielectric excited at approximately the same location. A large response gain is seen from 100 Hz to 600 Hz which is nearly sustained until approximately 1,500 Hz, at which point the magnitude of the response being to decrease. The response of the fluid free capacitor is much larger than the fluid filled capacitor, which indicates that though the amount of liquid dielectric in the container is relatively small, the damping added to the system by the fluid is significant.
Figure 61. Comparison of the Response for the Capacitor With and Without Liquid Dielectric

After finishing the tests described above, the failed capacitor was dissected, as discussed in Chapter 3. Once the internal elements had been removed, the shell was analyzed. The test setup was similar to that shown in Figure 59, using the mirrors to measure the response of the narrow side from excitation near the center of the broad side. For this test, the shell was painted with a reflective silver coating to improve the laser reflectance and, therefore, improve the measurements. Separate tests were run for each side of the shell. For both tests, the excitation for these tests was swept from 10 Hz to 2 kHz using a sweep rate of 60 seconds. Figure 62 shows the measured response measured of the broad side and Figure 63 shows the measured response of the narrow side. For the broad side, the largest peaks occur at 93 Hz, 210 Hz, 264 Hz, 328 Hz, 430 Hz, and 530 Hz, but there are many other resonant frequencies seen as well. The narrow side has resonant frequencies near those for the broadside, but the dominant frequencies are different. During this test, standing mode shapes were measured by the laser vibrometer. Some of these mode shapes are shown in Figure 64 for the broad side and Figure 65 for the narrow side. Like the FEA results, some mode shapes appeared that were not expected, but most had the predicted
half wave pattern. For the broad side, the half wave mode shapes fall within the limits set in Table 2 and Table 4 until 531 Hz. After that, the mode shapes appear before that range, but are still within 10% of the minimum values. The narrow side is not compared because it has one free edge.

Figure 62. Average Frequency Response for the Shell Broad Side

Figure 63. Average Frequency Response for the Shell Narrow Side
Figure 64. Experimental Mode Shapes for the Broad Side of the Shell
83 Hz (i=1,j=1) 276 Hz (i=1,j=3) 350 Hz (i=1,j=4)

430 Hz (i=1,j=5) 560 Hz (i=1,j=6) 668 Hz (i=1,j=7)

783 Hz (i=2,j=4) 972 Hz (i=2,j=5) 1,026 Hz (i=2,j=6)

Figure 65. Experimental Mode Shapes for the Narrow Side of the Shell
Validation

To check the results of the sweep test, the capacitor was excited using a sinusoidal input of a single frequency and the response was measured. Depending on the response time of the system and excitation sweep rate, the system may not have time to respond to the input. Comparing the sweep response with the response of a single, sustained excitation was used as a quick check to determine the validity of the sweep tests. Several tests were run using different frequencies of excitation. The frequencies and associated responses are given in Table 13. Figure 66 shows the sweep response along with the single frequency responses. The single frequency response seems to match the sweep response well at peaks, but there is some error between peak responses. However, because the response peaks are of primary interest, the sweep test results are adequate. Testing the structure with a single frequency also shows that, under the test conditions, the system is linear because the input and output frequencies are the same.

Table 13
Sine and Sweep Response Comparison

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Sine Response (mm/s/lbf)</th>
<th>Sweep Response (mm/s/lbf)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>735</td>
<td>1.58</td>
<td>1.58</td>
<td>0.2%</td>
</tr>
<tr>
<td>1,140</td>
<td>1.52</td>
<td>1.52</td>
<td>0.2%</td>
</tr>
<tr>
<td>1,150</td>
<td>1.53</td>
<td>1.53</td>
<td>0.2%</td>
</tr>
<tr>
<td>1,260</td>
<td>1.70</td>
<td>1.38</td>
<td>18.6%</td>
</tr>
<tr>
<td>1,320</td>
<td>1.72</td>
<td>1.45</td>
<td>15.9%</td>
</tr>
<tr>
<td>1,500</td>
<td>1.49</td>
<td>1.49</td>
<td>0.2%</td>
</tr>
<tr>
<td>1,920</td>
<td>1.42</td>
<td>1.26</td>
<td>11.2%</td>
</tr>
<tr>
<td>2,000</td>
<td>1.30</td>
<td>1.30</td>
<td>0.1%</td>
</tr>
<tr>
<td>2,140</td>
<td>1.27</td>
<td>1.22</td>
<td>3.2%</td>
</tr>
<tr>
<td>2,750</td>
<td>1.40</td>
<td>1.32</td>
<td>5.5%</td>
</tr>
<tr>
<td>3,340</td>
<td>1.22</td>
<td>1.15</td>
<td>5.7%</td>
</tr>
</tbody>
</table>
Adjusting the sweep rate was also used to check the validity of the sweep tests. The sweep rate is the amount of time taken to sweep the frequency excitation over the measured frequency range. Again, this was done to ensure that the system had time to respond to each excitation frequency. However, this was only done for some of the tests because the test time required is significantly longer. For the tests, the sweep rate was increased by a factor of 15. Also, for the slow sweep rate tests, the frequency band was set to 1 kHz to 2 kHz. This had to be done because running the tests over the 100 Hz to 4 kHz band caused the tests to fail due to lack of system memory. Figure 67 shows the response of the broad side of the capacitor to both sweep rates from 1 kHz to 2 kHz. The figure shows that despite having a considerably different sweep rate, the responses of both tests are approximately the same. This means that the fast sweep rate is adequate for the tests.
For all of the vibration tests, the capacitor was excited at one point and the response was measured at several points using the laser vibrometer. The Polytec software uses the response at each point along with the input force to determine the transfer function at each point. From these individual transfer functions, the software calculates the average system frequency response. In contrast, the test method used by Cox and Guan involved striking the test capacitor at several points and measuring the response at a single point. The transfer function at each impact point was determined and the system transfer function was determined by summing the individual transfer functions. According to the Maxwell’s reciprocal theorem, the frequency response measured at a point, A, resulting from excitation at another point, B, is equal to the frequency response measured at B resulting from excitation at A [10]. This means that Cox and Guan’s tests are equivalent to one of the tests run with the laser vibrometer. The theorem can be applied if the inputs and outputs are not mixed and the system is linear [8]. Because all inputs for the tests are forces and all outputs for the tests are velocities and the system is linear, the theorem holds.
**Results**

Using the average responses displayed in Figure 52 and Figure 56, the magnitude of the mobility measured at the harmonics of 60 Hz was determined. The response at the harmonics of 60 Hz is shown in Figure 68 for both the broad side and narrow side of the capacitor. The peak response for the broad side was measured at the 24\textsuperscript{th} harmonic, which is one of the frequencies of dominant acoustic emission. Peaks in the response are also seen at the 6\textsuperscript{th}, 13\textsuperscript{th}, 16\textsuperscript{th}, and 20\textsuperscript{th} harmonics. There is also little response seen at the first three harmonics. For the narrow side, peaks were measured at the 1\textsuperscript{st}, 10\textsuperscript{th}, 13\textsuperscript{th}, and 16\textsuperscript{th} harmonics. Both the 22\textsuperscript{nd} and 24\textsuperscript{th} harmonics are in a region of low response.

![Figure 68. Comparison of the Average Response for the Narrow Side and Broad Side](image)

Comparing the two sides, the peak response per input on the broad side is approximately seven times as large as the peak response on the narrow side. This leads to the conclusion that though much larger excitations occur on the narrow sides, the velocity of the shell is the largest
on the broad side. This is clear from Figure 69, which shows the percent increase in response for the broad side compared to the narrow side at each harmonic.

![Figure 69. Increase in Frequency Response for the Broad Side](image)

**Conclusions**

Vibration analysis was performed on the two capacitors provided by The local utility provider using a scanning laser vibrometer. The working capacitor was extensively tested using multiple excitation locations on both the broad sides and narrow sides. From the acoustic measurements given in Chapter 2, the frequencies of dominant acoustic emission at SUB 1 are the $2^{\text{nd}}$, $4^{\text{th}}$, $8^{\text{th}}$, $14^{\text{th}}$, $22^{\text{nd}}$, and $24^{\text{th}}$ harmonics of 60 Hz. At SUB 2, the dominant acoustic frequencies are the $2^{\text{nd}}$, $4^{\text{th}}$, $6^{\text{th}}$, and $12^{\text{th}}$ harmonics of 60 Hz. The average frequency response of the broad side of the capacitor had response peaks at the $6^{\text{th}}$, $9^{\text{th}}$, $13^{\text{th}}$, $20^{\text{th}}$ and $24^{\text{th}}$ harmonics. Only the $24^{\text{th}}$ harmonic had the response that was expected. However, when the broad side was excited near the center, peaks in the response were seen at the $8^{\text{th}}$, $12^{\text{th}}$, $19^{\text{th}}$, and $22^{\text{nd}}$ harmonics. For the narrow side, peaks in the response were measured at 750 Hz, 950 Hz, and 1.6 kHz. There were no peak responses at the frequencies of dominant acoustic generation. The measurements
also showed that though the narrow sides see the largest excitation, the broad sides have the largest response.

The capacitor was also excited on both sides while measuring the response on the opposite side. Results from the tests showed that excitation on the narrow side has little effect on the broad side, but excitation the broad side caused a significant response on the narrow side. The capacitor without the dielectric fluid was also tested and showed that the dielectric fluid had a large damping effect on the system response. After this capacitor was dissected, the shell was also tested and the results matched those in the FEA analysis to within 10%.

To check the measurements, the response from a sweep excitation was compared to the response of the capacitor to a single frequency. The results matched well when the single frequency was near a response peak, but some frequencies that were not near peaks had significant error. The sweep rate was also adjusted to ensure the system had adequate time to fully respond. The tests showed that the sweep rate had little effect on the system, most likely due to the system being heavily damped. It was also determined by Maxwell’s reciprocity theorem that the impact hammer testing done by Cox and Guan is equivalent to the methods used with the scanning laser vibrometer.
Banks of capacitors at two substations that feed a large electric arc furnace are creating audible noise as a result of exposure to voltage squared harmonics. Through acoustic field measurements, it was determined that the frequencies of dominant acoustic emission are the 2nd, 14th, 22nd, and 24th harmonics of 60 Hz at SUB 1 and the 2nd, 4th, 6th, and 12th harmonics of 60 Hz at SUB 2. Of these frequencies, the largest magnitude occurs at the 22nd and 24th harmonics at SUB 1 and the 2nd and 4th harmonics at SUB 2 [11]. However, the magnitude of the current near these frequencies is less than that seen at other frequencies. It was also noticed that the acoustic frequencies are at 60 Hz offsets of the current frequency. This behavior was verified in a similar study found through a literature review [5].

For this study, the local utility provider provided two shunt capacitors similar to those used at SUB 1 and SUB 2 for use as test specimens. One capacitor had failed in service and permission was given to dissect it. This was done to determine the construction of the shell as well as the construction and orientation of the internal elements. Through the dissection, it was determined that the shell of the capacitor is made of relatively thin (1/16 inch) stainless steel and has no extra internal support. Regarding the internal elements, there were sixteen capacitive elements wired in a series/parallel configuration. The elements are constructed by winding layers
of aluminum foil and polypropylene. The entire group is then impregnated with a liquid
dielectric. The elements are arranged vertically within the shell and are tightly packed, but not
directly constrained. Measurements of the layers were taken and confirmed through a paper
found during the literature review.

After dissecting the capacitor, the internal components were analyzed to determine how
the electric forces generated within the elements excite the structure. When the elements are
energized, the forces generated by the attraction of the positively charged plate to the negatively
charged plate act to contract the layers, squeezing out the liquid dielectric. As the elements
expand to a relaxed state, they either directly contact the shell or create a pressure wave in the
liquid dielectric that excites the shell. It was determined that the maximum pressure generated
occurs at the outermost layer of each element. The limit to the maximum pressure was calculated
to be approximately 8 psi. It was also determined that the 60 Hz offset seen in the acoustic data is
a result of the forces generated being a function of the voltage squared. This causes the excitation
frequency to be double the current frequency.

One of the original goals of the project was to determine the natural frequencies of the
capacitor through theoretical methods. However, because of the complexities of the system and
time limitations, this was not done. FEA software was used to determine the natural frequencies
and mode shapes of the shell. Through the modeling, it was determined that the shell has several
natural frequencies near the harmonics of 60 Hz. It was also determined that most of the
corresponding mode shapes take on a half wave pattern typical of simply supported and clamped
plates. The results of the modeling matched the experimental results to within 10%. This analysis
serves as a good starting point for any further modeling that might be done.
Experimental vibration analysis of the capacitor was done using a scanning laser vibrometer. From the test data, it was seen that the narrow side of the capacitor has no response peaks in the frequency range of interest. The results of the broad side tests did show response peaks at some of the frequencies of acoustic generation. It was also determined that the magnitude of the response on the broad side is the largest, even though the excitation on the narrow side is dominant. One of the benefits of using the laser vibrometer system is the ability to experimentally determine mode shapes. During all of the tests on the working capacitor, no standing mode shapes were seen. All of the waves generated propagate towards the edges and die out. This is most likely due to the system being very heavily damped.

Putting together all of the information gathered, it is clear that the system is still not fully understood. Examining Figure 25, it can be seen that the excitation at the acoustic harmonics is much smaller than at the fundamental. Though there are peaks in the response at some of the frequencies of acoustic emission, the gains needed at these frequencies are not present. Because extensive vibration analysis was performed on the bank, it is believed that follow up work is needed regarding the magnitude of the excitation generated.

Possible Solutions

Though the root cause of the acoustic generation was not completely addressed, there are some solutions that could reduce the magnitude of the audible noise. These options include filtering the current harmonics, changing the systems mechanical natural frequencies, and installing acoustic enclosures around the banks. There are also solutions that could be implemented by the manufacturer if the capacitors were to be redesigned.
**Filters**

An obvious solution would be to filter out the current harmonics. This was investigated by Haskew during his study. Installing filters at both the point of injection (the mill) and at the capacitor banks was investigated. Haskew determined that this would be a very difficult option to implement. The load drawn by the mill is sometimes irregular, resulting in time varying harmonic content. At the capacitor banks, the resonant electrical frequencies also change as a result of switching banks online and offline in different configurations, which is illustrated in Figure 70. The magnitude of the current injections at the higher harmonics is also small, which would make these harmonics difficult to filter. Because of the high implementation cost of the filters and the uncertainty of their effect, filtering the harmonics was not recommended.

![Figure 70. Change in System Electrical Resonance Resulting from Capacitor Switching](image)
Shifting Mechanical Natural Frequencies

Another possible solution may be to shift the mechanical natural frequencies away from the problem acoustic frequencies. This could be done by either increasing the stiffness on the sides of the capacitor, which would shift the natural frequencies higher, or increasing the mass of the capacitor, which would shift the natural frequencies lower. From Figure 52 and Figure 56, it appears that the natural frequencies would need to be shifted lower in order to decrease the magnitude of the response. However, this may push other natural frequencies into the frequencies of excitation.

Acoustic Enclosures

A third option is to install an acoustic enclosure around the capacitor. This would modify the acoustic transmission path and increase the level of transmission loss. It is envisioned that an acoustic enclosure could be created using an inner layer of sound reducing material covered by an outer layer of stainless steel. Table 14 shows the reduction in acoustic magnitude that can be achieved at each octave band through this kind of enclosure [3]. As shown, these acoustic enclosures work well at each octave. Through a literature review, this was verified in a study found in which the noise generated by a transformer was reduced by 22-52 dB using an acoustic enclosure [1].

<table>
<thead>
<tr>
<th>Panels of Sandwich Construction</th>
<th>Thickness (mm)</th>
<th>Weight (kg/m²)</th>
<th>Octave Band Center Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>16 gauge steel and 100 mm of glass fiber</td>
<td>100</td>
<td>25</td>
<td>63</td>
</tr>
<tr>
<td>As above, covered by 22 gauge perforated steel</td>
<td>100</td>
<td>-</td>
<td>25</td>
</tr>
<tr>
<td>As above, but 16 gauge steel replaced with 5mm steel plate</td>
<td>100</td>
<td>50</td>
<td>31</td>
</tr>
</tbody>
</table>
Redesign the Capacitor

There are also some possible solutions available if the capacitors were to be redesigned. It was shown in Chapter 5 that the dielectric fluid has a large damping effect, which reduced the magnitude of the response by 40% between 500 Hz and 1.5 kHz. Adding more dielectric fluid to the capacitor will likely further reduce the magnitude of the response. Also, after dissecting the test capacitor, it was determined that the internal elements are only constrained by the shell. It is recommended that the elements be strapped together and not secured so that they do not contact the shell.

Recommended Solution

Install an acoustic enclosure around the individual capacitors at both substations. The acoustic enclosure should be made of 16 gauge stainless steel surrounding 100 millimeters (approximately 4 inches) of glass fiber. This will reduce the magnitude of the audible noise generated by 20 to 48 dB. The current and estimated resulting noise level after implementation is shown in Table 15. For SUB 1, the reference sound pressure used is not the standard sound pressure reference.

<table>
<thead>
<tr>
<th>Octave (-)</th>
<th>SUB 1 With Enclosure (dB)</th>
<th>SUB 2 With Enclosure (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>63</td>
<td>-55</td>
<td>10</td>
</tr>
<tr>
<td>125</td>
<td>-46</td>
<td>50</td>
</tr>
<tr>
<td>250</td>
<td>-52</td>
<td>65</td>
</tr>
<tr>
<td>500</td>
<td>-47</td>
<td>33</td>
</tr>
<tr>
<td>1,000</td>
<td>-31</td>
<td>30</td>
</tr>
<tr>
<td>2,000</td>
<td>-41</td>
<td>18</td>
</tr>
</tbody>
</table>

Table 15
Estimated Noise Reduction with Acoustic Enclosure

96
**Future Work**

It was originally planned that measurements be made at the substations while the banks were generating acoustic noise using the laser vibrometer. However, these plans were not approved due to safety concerns. This is still considered the best option to determine how the capacitors are vibrating when exposed to current harmonics and should be looked into further. More work is also needed in determining the capacitor’s natural frequencies using theoretical methods. It would also be beneficial to model the movement of the plates due to the generated electric forces using FEA software. Additional work is also needed in combining the results for the excitation and mechanical response with acoustic theory. If the acoustic enclosures were implemented, it would be beneficial to take additional acoustic measurements at both substations to determine their effect.
REFERENCES


